ELEMENTARY MATHEMATICS

PART II

ELEMENTARY MATHEMATICS

SPECIALLY PREPARED FOR CENTRAL SCHOOLS, SENIOR ELEMENTARY SCHOOLS, AND UPPER STANDARDS (VI, VII, VIII)

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PART II

(*for Pupils of* 12–14 *years*)

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PLAN OF THE BOOK

There are three sections, namely: (i) Arithmetic related to commerce; (ii) Algebra; and (iii) Geometry and Mensuration related to technology and leading to more advanced work in Mathematics, *e.g.* Logarithms, Trigonometry, and Deductive Geometry.

The special aims of the authors are:

1. To ensure that the ground-work of mathematics is well prepared.
2. To make the work as comprehensive as possible at this stage.
3. To graduate the exercises to be worked by the pupils.
4. To use graphic methods for the purpose of assisting the young mind to comprehend the subject.
5. To proceed from particular cases to the general statement.
6. To interest the pupils by reference to their work and to their daily experience.

The authors wish to express their indebtedness to the Council of the Union of Lancashire and Cheshire Institutes for kind permission to use examination questions.

CONTENTS

SECTION I

Arithmetic

SECTION II

Algebra

SECTION III

Geometry and Mensuration

Elementary Mathematics

SECTION I

ARITHMETIC

Revision—The Four Rules

EXERCISE 1

1. Complete the following table, showing import and export values for six months of the year 1925 :

1925.	Imports.	Exports.	Excess of Imports.
	£	£	£
April .	110,358,208	73,287,548	. . . .
May . .	104,278,466	78,703,466	. . . .
June .	110,982,155	69,493,391	. . . .
July . .	98,744,849	76,202,599	. . . .
Aug. .	91,732,604	74,537,667	. . . .
Sept. .	97,925,034	71,260,698	. . . .
Totals	. . . .	. . . .	. . . .

2. Complete the following table by additions :

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
784	17	$9\frac{1}{2}$	468	18	$10\frac{1}{4}$	386	14	$11\frac{1}{2}$	189	15	$8\frac{3}{4}$	. . . .		
276	18	$10\frac{1}{4}$	976	9	$11\frac{1}{2}$	785	17	$8\frac{1}{4}$	446	14	$9\frac{1}{2}$	. . . .		
493	15	$8\frac{3}{4}$	584	12	$8\frac{3}{4}$	673	10	$7\frac{3}{4}$	385	7	$4\frac{1}{2}$	. . . .		
287	19	$2\frac{1}{4}$	395	15	$7\frac{1}{2}$	918	17	$9\frac{1}{2}$	497	16	$5\frac{1}{4}$	. . . .		
985	14	$8\frac{1}{2}$	588	18	$6\frac{3}{4}$	279	16	$3\frac{1}{2}$	885	8	$6\frac{1}{2}$	. . . .		
. . . .			. . . .			. . . .			. . . .			. . . .		

1

3. How much is the sum of 800 dozen, 350 score and 560 gross, short of a million ?

4. Find, to the nearest penny, the average daily income of a man receiving £250 per annum.

5. The various departments of a store received £368, 15s. 6d., £1849, 13s. 11d., £1680, 15s. 9d., £209, 11s. 8d., £459, 19s. 7d., £1765, 18s. 5d., respectively. How much are the total receipts less than £10,000 ?

6. By how much does the sum of £7643, 19s. $11\frac{1}{4}$d. and £10,985, 15s. $5\frac{1}{2}$d. exceed their difference ?

7. A motorist starts from a place 120 ft. high ; he ascends 560 ft., then descends 250 ft., climbs 925 ft., and then descends 750 ft. At what height above sea-level is he then ?

8. On Jan. 1 the sun rises at London at 8.6 a.m. and sets at 4.1 p.m. ; on July 1 the sun rises at London at 3.47 a.m. and sets at 8.20 p.m. Find the difference in the period of daylight on these dates.

Short Methods

Exercise 2

Work the following by the quickest methods you know :

1.	(*a*) 8769×10	(*b*) 735×100	(*c*) 859×1000	(*d*) $38 \times 10{,}000$
2.	(*a*) 7846×9	(*b*) 8375×99	(*c*) 2897×999	(*d*) 3371×98
3.	(*a*) 6925×398	(*b*) 7098×596	(*c*) 2354×695	(*d*) 764×797
4.	(*a*) 8912×25	(*b*) 3745×250	(*c*) 79×2500	(*d*) 1865×25
5.	(*a*) 706×625	(*b*) 485×625	(*c*) 469×375	(*d*) 875×875
6.	(*a*) $1832 \times 12\frac{1}{2}$	(*b*) $795 \times 37\frac{1}{2}$	(*c*) $58521 \times 16\frac{2}{3}$	(*d*) $9347 \times 7\frac{1}{7}$
7.	(*a*) $4976 \div 10$	(*b*) $12358 \div 100$	(*c*) $4681 \div 1000$	(*d*) $23579 \div 200$
8.	(*a*) $3758 \div 25$	(*b*) $12469 \div 125$	(*c*) $38495 \div 625$	(*d*) $78910 \div 375$

Approximation

Exercise 3

1. Write the following numbers, correct to the nearest hundred : 7510 ; 8458 ; 951 ; 3275 ; 6789 ; 2341.

2. Write the following numbers, correct to the nearest ten : 762 ; 585 ; 622 ; 451 ; 168.

3. Consider the number 43·852 and then write (*a*) the nearest whole number, (*b*) the nearest number correct to one decimal place, (*c*) the nearest number correct to two decimal places.

4. Give answers to the following, correct to two decimal places :

(*a*) $785{\cdot}9 \times {\cdot}37$.
(*b*) $482{\cdot}5 \times {\cdot}009$.
(*c*) ${\cdot}3846 \times 56$.
(*d*) $37{\cdot}84 \div {\cdot}9$.
(*e*) ${\cdot}323 \div 2{\cdot}7$.
(*f*) $1 \div {\cdot}19$.

5. Find, to the nearest penny, the average price per lb. of rubber when 845,600 lbs. are sold for £115,384.

6. Find, to the nearest farthing, the cost per lb. of sugar when 3,584,620 cwts. are sold for £5,817,293.

7. Find, to the nearest whole number, the average number of tram journeys made per person per week in a certain town having a population of 248,320, if the total number of tram journeys made per year amounted to 38,584,372.

8. The length of a certain railway is 45,764 miles, and the gross receipts for a year are £90,317,896. Find, to the nearest shilling, the average receipts per week per mile.

9. If $\dfrac{1}{x} = 0{\cdot}357$, find the value of x correct to three decimal places.

10. (*a*) Find the cubical contents of 1000 galls. to the nearest cubic foot when told that 1 gall. $= 277{\cdot}4$ cu. ins.

(*b*) Find, to the nearest gallon, the capacity of the water that will fill a cubical tank one side of which is $10{\cdot}5$ ft. (1 cu. ft. $= 6{\cdot}23$ galls.)

Fractions

Exercise 4

1. A motor lorry travels $31\frac{5}{8}$ miles in $2\frac{3}{4}$ hrs. A second lorry travels $54\frac{3}{16}$ miles in $5\frac{2}{3}$ hrs. Find the difference in their average speed per hour.

2. A man who spends ·75 of his income every week saves £28·6 in a year of 52 weeks. Find his weekly income.

3. When a bullet weighs ·27 ozs., how many bullets can be made from 2 lbs. of lead ? What weight remains ?

4. In a school $\frac{1}{5}$ of the pupils are Infants, $\frac{3}{8}$ of the remainder are Juniors, and 180 are Seniors. How many pupils are there in the school ?

5. Of 27,500 persons visiting a seaside resort ·44 of the whole held season tickets, ·38 held day tickets, and the remainder held half-day tickets. How many were there of each ?

6. At an election 8960 people voted for one or other of two candidates, and the elected candidate had a majority of 1020 votes. What fraction of the total votes did each receive ?

7. Calculate, to the nearest minute, the length of a year which is said to be 365·242218 days.

8. A pile of 8 reams of paper is 64·8 ins. in height. Find, to two decimal places, the thickness of one sheet of paper. (1 ream = 480 sheets.)

9. If $\frac{1}{8}$ of a telegraph pole is cut off, the remainder is 6 yds. 2 ft. long. What would have been the length of the remainder if $\frac{9}{16}$ of the pole had been cut off ?

10. If $6\frac{2}{3}$ lbs. of tea cost 16s. 8d., find the cost of $4\frac{1}{4}$ lbs.

11. If 3 metres = 3·2809 yds., express a centimetre in inches.

12. A boy travels on a railway at half fare. For a journey of 107 miles his father pays £1, 6s. 9d. for tickets for both of them. How far could they both travel at the same rate for 5s. 3d. ?

13. A coal merchant sells 3 tons 8 cwts. 34 lbs. daily, and another sells 5 tons 11 cwts. 108 lbs. daily. What fraction is the first weight of the second ?

14. A sample of 10 c.c. of air contains 7·83 c.c. of nitrogen, 2·14 c.c. of oxygen, and other gases. Express the ratio of the volume of oxygen to the volume of nitrogen as a decimal fraction correct to two decimal places.

15. At the first stopping place $\frac{1}{3}$ of the passengers in a tram-car leave and 20 people enter : at the next stopping place $\frac{1}{18}$ of the new total dismount and 5 people enter. The car now carries 56 passengers. How many people were there in the car at first ?

16. What is the height of a pile of 58,560 newspapers if each consists of 4 sheets and the thickness of each sheet is ·012 in. ? Give the answer in yards, feet, and inches to the nearest inch.

17. When the difference between the circumference and the diameter of a circle is 25 ft., find the diameter. ($\pi = \frac{22}{7}$.)

18. Express, as the fraction of an acre, the area of a rectangular field $52\frac{1}{2}$ yds. by 80 yds.

19. Find the cost in francs of 30·5 metres of silk at 20·55 francs per metre.

20. Express 5 Kg. 5 Dg. as decigrams.

Ratio, Proportion, and Simple Interest

Exercise 5

1. When a man allows a debtor a discount of 1s. $1\frac{1}{2}$d. in the pound, he receives £755 instead of the full amount. What was the full amount ?

2. When 28 men earn £168, 6s. 8d. by working 9 days, how long must 84 boys work to earn the same amount if each boy's wage is $\frac{2}{3}$ that of a man's ?

3. A shopkeeper buys sugar at 37s. 4d. per cwt. What must be his selling price per lb. if he is to make a profit of 20 per cent. on his *selling* price ?

4. A builder makes a profit of 20 per cent. on his cost price by selling a house to an agent for £750, and the agent sells it to a customer for £850. What percentage profit on his cost price does the agent make ? What would the builder's percentage profit on his cost price have been if he had sold direct to the customer for £850 ?

5. Calculate, to the nearest penny, the simple interest on £375, 17s. for 85 days at 5 per cent. per annum.

6. How long will it take 51 horses to plough 1296 acres if 34 horses plough this area in 35 days ?

7. A man owning £9000 invests $\frac{2}{3}$ of it at 4 per cent. and the rest at 5 per cent. Find his half-yearly income from these investments.

8. A bankrupt who owes £1075 can only pay £340. What is the ratio of a creditor's account to the money he actually receives ?

9. Find, to the nearest penny, the amount of £987, 11s. 8d. for 3 years 6 months at $4\frac{1}{2}$ per cent. per annum simple interest.

10. A man invests £872 at 6 per cent. per annum for 73 days, and calculates the interest to which he is entitled by multiplying the principal by the number of days and dividing by 6000. What error does he make ?

11. A man bought 10 German pianos, paying a duty on them of 25 per cent., and sold them at a loss of 5 per cent. on his total cost price. If he had sold the lot for £30 more, his profit would have been 1 per cent. What did the German manufacturer obtain for the pianos ?

12. A pile is driven into a river bed ; $\frac{1}{4}$ is in the mud,

$\frac{1}{3}$ is in the water, and 6 ft. 3 ins. is above the water. If the pile is driven half as far again into the mud, how much now remains above the water?

13. During a sale a cycle dealer allows 20 per cent. off the marked price of each cycle. His profit is then 15 per cent. on his outlay. At what price must he mark a cycle which cost him £6?

14. £150, 8s. 0d. is invested for 3 years at $2\frac{1}{2}$ per cent. per annum. What is the total amount that will be drawn at the end of this time if simple interest is paid?

15. If tea cost 2s. 6d. a lb., for what sum must 1 cwt. be sold so as to make a profit of $12\frac{1}{2}$ per cent. on the cost price?

16. A lady left £12,984, 3s. 4d. to be divided between her daughters in the ratio 2 : 3. Find each daughter's share.

17. Divide £5. 5s. 0d. between A and B, so that when A gets 17s. 6d., B gets 12s. 6d.

18. What sum must be paid to insure a cargo of £500 at $3\frac{1}{2}$ per cent., so that in case of loss both the value of the cargo and premium may be recovered?

19. Of 54,000 voters, ·37 voted for one candidate and ·28 for the other. How many did not vote?

20. If 415 galls. of spirit is diluted with 115 galls. of water, find the percentage of spirit and of water in the mixture.

The Metric System of Weights and Measures

The Metric System is so called from the metre (French, *mètre*).

Metre—*the unit of length*; supposed to be equal to $\frac{1}{10,000,000}$ part of the earth's quadrant.

Litre—*the unit of capacity*; the volume of a cubic decimetre of water at 4° C.

Gram—*the unit of weight*; the weight of a cubic centimetre of water at 4° C.

For ordinary purposes the Kg. is treated as the unit of weight.

Are—*the unit of surface*; a square Dekametre, *i.e.* 100 sq. metres.
Stere—*the unit of solidity*; a cubic metre.

The relation of the various units is shown in figure 1.

The Metric Tables

Kilo = 1000	Hekto = 100	Deka = 10	Unit = 1	deci- = $\frac{1}{10}$	centi- = $\frac{1}{100}$	milli- = $\frac{1}{1000}$
Kilo-metre.	Hekto-metre.	Deka-metre.	Metre.	deci-metre.	centi-metre.	milli-metre.
Kilo-litre.	Hekto-litre.	Deka-litre.	Litre.	deci-litre.	centi-litre.	milli-litre.
Kilo-gram.	Hekto-gram.	Deka-gram.	Gram.	deci-gram.	centi-gram.	milli-gram.

100 Square metres = 1 Are.
100 Kilograms = 1 Quintal.
1000 Kilograms = 1 Tonne.
1 Kilolitre = 1 Cubic metre = 1 Stere.
1 Litre = 1000 Cubic centimetres.

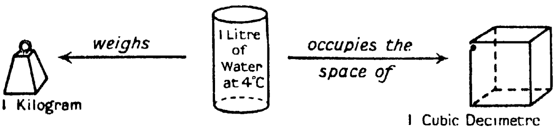


FIG. 1.

Approximate Equivalents

Linear measure	1 cm. = 0·4 ins.		1 in.	25·4 mm.
	1 metre = 39·37 ins.		1 yd.	0·914 metre.
	1 Km. = 0·62 miles		1 mile	1·6 Km.
Square „	1 sq. metre = 1·2 sq. yds.		1 sq. in.	6·45 sq. cm.
	1 Are = 119·6 sq. yds.		1 acre	0·4 Hektare.
Cubic „	1 c.c. = 0·06 c. in.		1 cu. in.	16·4 c.c.
Capacity	1 litre = 1·76 pints		1 gall.	4·5 litres.
Weight	1 Kg. = 2·2 lbs.		1 lb.	0·45 Kg.

EXERCISE 6

[Use the approximate equivalents given above, and give answers correct to two decimal places only.]

1. A cyclist travels 500 m. a minute. Express this speed in miles per hour.

2. Find the number of litres in a cubic foot.

3. What is the acreage of a field 127·3 m. by 137·8 m. ?

4. Find, in gallons, the amount of water in a tank 31·3 dm. long and 29·2 dm. wide, if the water is ·571 m. in depth.

5. A merchant buys 1800 Kg. of metal for £51, 9s. 0d. For what must he sell it to gain 1d. per lb. ?

6. The speed of an aeroplane is 75 miles per hour. Find its speed in metres per second.

7. The diameter of a halfpenny is 1 in., and of a franc 2·56 cm. By how many centimetres does the distance covered by 398 francs exceed that covered by 398 halfpennies when the coins are laid in a row, flat and edge to edge ?

8. A block of granite was ordered by a French firm from an English quarry. The order was given for a block 4 m.×4 m.×1 m., but the mason cut the block 4 yds.× 4 yds.×1 yd. How many c.c.'s was the block too small ?

9. On a long march a soldier must carry a pint of water for each 40 miles. How many litres must he carry on a march of 49 Km. ?

10. What is the diameter in inches of a circle whose circumference is 10·41 m. ? $\pi = 3 \cdot 14$.

11. To what depth in cm. is a skating rink flooded if its area is 1 acre and 500,000 litres of water are contained in it ?

12. A concrete pipe has an internal diameter of 3·5 m., and it is 800 m. in length. How many gallons of water does it hold when it is full ? $\pi = 3 \cdot 14$.

13. How many c.c.'s. of wood are there in a closed box whose external dimensions are 12·7 ins.×6 ins.×3·3 ins., and internal dimensions are 12·1 ins.×5·6 ins.×2·9 ins. ?

14. A piece of masonry 28·5 yds.×13 yds.×6·2 yds. is lowered into a rectangular reservoir 98 m.×159 m., in which the water is 44 m. deep. Find, to the nearest centimetre, how much the water will rise.

15. I was weighed in England before my holiday, and weighed 8 stone. After a month in Switzerland I weighed 51 Kg. By how many lbs. had my weight increased ?

16. A new railway in South America was 120 Km. long. A British firm contracted to build it for £537,500. Find the cost per mile in English money to the nearest sovereign.

17. What is the weight in Kg. of 3·5 cm. of rain on an area of 2500 sq. m. ?

18. A tank is 5·5 m. long and 4·4 m. wide. When it contains 1230 litres, how deep is the water in the tank ?

Decimalisation and De-decimalisation of Money

Shillings, Sixpences, Threepences, Ninepences

The pupil must remember that

1s. = £·05 ; 6d. = £·025 ; 3d. = £·0125 ; 9d. = £·0375.

Therefore, when decimalising we may express:

(i) Shillings as hundredths of £1 by multiplying by 5,

$$e.g.\ 17\text{s.} = £17 \times {\cdot}05 = £{\cdot}85 \left(\text{or } 17\text{s.} = \frac{£17 \times 5}{100} = £{\cdot}85\right).$$

(ii) Amounts such as 9s. 6d., 13s. 6d., as hundredths of £1 by multiplying the shillings by 5 and adding £·025,

$$e.g.\ 13\text{s. } 6\text{d.} = £13 \times {\cdot}05 + £{\cdot}025 = £{\cdot}675,$$

$$\text{or } 13\text{s. } 6\text{d.} = \frac{£13 \times 5}{100} + £{\cdot}025 = £{\cdot}675.$$

(iii) Amounts such as 7s. 3d., 11s. 3d., as hundredths of £1 by multiplying the shillings by 5 and adding £·0125,

$$e.g.\ 7\text{s. } 3\text{d.} = £7 \times {\cdot}05 + £{\cdot}0125 = £{\cdot}3625,$$

$$\text{or } 7\text{s. } 3\text{d.} = \frac{£7 \times 5}{100} + £{\cdot}0125 = £{\cdot}3625.$$

(iv) Amounts such as 3s. 9d., 14s. 9d., as hundredths of £1 by multiplying the shillings by 5 and adding £·0375,

$$e.g.\ 3\text{s. } 9\text{d.} = £3 \times {\cdot}05 + £{\cdot}0375 = £{\cdot}1875,$$

$$\text{or } 3\text{s. } 9\text{d.} = \frac{£3 \times 5}{100} + £{\cdot}0375 = £{\cdot}1875.$$

Conversely, we may de-decimalise such amounts by dividing the total number of hundredths by 5. This quotient represents the shillings, and the remainder indicates the pence,

$$e.g.\ £{\cdot}35 = \frac{35\text{s.}}{5} = 7\text{s.}$$

£·375 = 7s.	£·025 = 7s. 6d.
£·3625 = 7s.	£·0125 = 7s. 3d.
£·3875 = 7s.	£·0375 = 7s. 9d.

Exercise 7 (*Revision*)

1. Express as a decimal of £1 :

(*a*)	£5, 10s. 0d. ;	£7, 15s. 0d. ;	£3, 5s. 0d. ;	£1, 14s. 0d. ;	£2, 18s. 0d.
(*b*)	£8, 4s. 0d. ;	£5, 6s. 0d. ;	£1, 16s. 0d. ;	£3, 2s. 0d. ;	£2, 12s. 0d.
(*c*)	£1, 1s. 0d. ;	£3, 3s. 0d. ;	£5, 5s. 0d. ;	£7, 7s. 0d. ;	£9, 9s. 0d.
(*d*)	£11, 11s. 0d. ;	£19, 19s. 0d. ;	£17, 17s. 0d. ;	£13, 13s. 0d. ;	£8, 8s. 0d.

2. Express as a decimal of £1 :

(*a*)	2s. 6d. ;	7s. 6d. ;	12s. 6d. ;	17s. 6d. ;	6d.
(*b*)	£1, 19s. 6d. ;	£2, 18s. 6d. ;	£3, 16s. 6d. ;	£1, 15s. 6d. ;	£5, 14s. 6d.
(*c*)	£6, 13s. 6d. ;	£7, 11s. 6d. ;	£6, 10s. 6d. ;	£5, 9s. 6d. ;	£4, 8s. 6d.
(*d*)	6s. 6d. ;	5s. 6d. ;	4s. 6d. ;	3s. 6d. ;	1s. 6d.

3. Express in £, s. d. :

(*a*) £0·5 ;	£0·25 ;	£0·75 ;	£0·325 ;	£0·875
(*b*) £0·625 ;	£0·375 ;	£0·125 ;	£2·55 ;	£2·8
(*c*) £1·2 ;	£2·3 ;	£3·95 ;	£1·1 ;	£4·9
(*d*) £3·15 ;	£8·4 ;	£2·85 ;	£6·6 ;	£7·7

4. Express in £, s. d. :

(*a*) £2·025 ;	£3·975 ;	£1·675 ;	£3·325 ;	£1·925
(*b*) £3·575 ;	£2·275 ;	£3·825 ;	£3·775 ;	£3·075
(*c*) £4·525 ;	£3·475 ;	£2·425 ;	£0·225 ;	£0·175
(*d*) £8·8 ;	£9·9 ;	£10·1 ;	£12·2 ;	£9·75

Exercise 8

1. Express as a decimal of £1 :

(*a*) £3, 5s. 3d. ;	£4, 7s. 9d. ;	£2, 13s. 3d. ;	£1, 19s. 9d.
(*b*) £4, 11s. 9d. ;	£2, 1s. 3d. ;	£2, 18s. 9d. ;	£5, 15s. 3d.
(*c*) £1, 13s. 3d. ;	£7, 17s. 9d. ;	£9, 9s. 9d. ;	£2, 2s. 9d.
(*d*) £2, 6s. 9d. ;	£1, 17s. 3d. ;	£3, 16s. 3d. ;	£1, 18s. 3d.

2. Express in £, s. d. :

(*a*) £3·025 ;	£1·4125 ;	£3·725 ;	£0·9875
(*b*) £6·375 ;	£2·2875 ;	£1·9625 ;	£3·325
(*c*) £2·6875 ;	£1·3125 ;	£0·225 ;	£0·5125
(*d*) £3·2125 ;	£0·675 ;	£0·8375 ;	£0·4125

Approximate Results

$$\tfrac{1}{4}\text{d.} = £\tfrac{1}{960} = £\tfrac{1}{1000} + £\tfrac{1}{24000}.$$

If, then, we express $\frac{1}{4}$d. as £·001, the expression is too small by £$\frac{1}{24000}$.

We may use this fact thus :

Example. Express £3, 13*s.* 9$\frac{3}{4}$*d. as the decimal of* £1.

£3	= £3.
13s.	= £0·65.
9$\frac{3}{4}$d. = 39 f.	= £0·039.
	£3·689.

And this is too small by £$\frac{39}{24000}$, *i.e.* £·001625, *i.e.* 1·56 farthings.

If we study the errors occurring by the above method we can obtain a rule for decimalising any sum correct to three decimal places.

11 farthings	$=£·011\frac{11}{24}=£·011$	correct to 3 places.			
12	,,	$=£·012\frac{12}{24}=£·013$	,,	,,	,,
13	,,	$=£·013\frac{13}{24}=£·014$	,,	,,	,,

35 farthings	$=£·035\frac{35}{24}=£·036$	correct to 3 places.			
36	,,	$=£·036\frac{36}{24}=£·038$	,,	,,	,,
37	,,	$=£·037\frac{37}{24}=£·039$	,,	,,	,,

The rule follows, namely,

1. Express pounds and shillings as already shown.
2. Reduce pence and farthings to farthings and express these as thousandths of £1, **but,** correct by adding 1 for numbers above 11 and by adding 2 for numbers above 35. Add.

Example. Express £2, 13s. 10¾d. as the decimal of £1.

£2, 13s. 0d.	= £2·65
10¾d. = 43 f. (add 2) =	£0·045
	£2·695

Conversely, by studying the decimals £·013, £·014, £·038, £·039, we obtain a rule for de-decimalising to the nearest farthing, thus:

1. Divide the total number of hundredths to find the shillings.
2. Carry the remainder (if any), as in division, to the next figure. This gives the farthings, **but,** correct by subtracting 1 for numbers above 12 and by subtracting 2 for numbers above 37. Add.

Example. Express £3·942 as £, s. d. to the nearest farthing.

£3	= £3		
·94 ÷ 5	=	18	0
Remainder is 42 and (42 − 2) f. =			10
	£3	18	10

Exercise 9

1. Express as a decimal of £1 correct to three decimal places:

(*a*)	£1, 7s. 3¾d.;	£3, 6s. 5¼d.;	£2, 19s. 2½d.;	£8, 13s. 9½d
(*b*)	£2, 18s. 4½d.;	£1, 9s. 4¾d.;	£3, 13s. 6¾d.;	£12, 12s. 10¾d.
(*c*)	£3, 19s. 9¼d.;	£2, 18s. 8¼d.;	£7, 11s. 11½d.;	£3, 15s. 4½d.
(*d*)	18s. 6¾d.;	£5, 1s. 11½d.;	£6, 2s. 7¾d.;	£6, 3s. 10¼d.

2. Express as £, s. d. correct to the nearest farthing:

(*a*)	£1·879;	£0·777;	£0·111;	£2·379
(*b*)	£0·123;	£2·936;	£1·270;	£3·666
(*c*)	£0·567;	£3·818;	£5·385;	£0·808
(*d*)	£0·389;	£5·325;	£2·477;	£1·998

3. Find the price of 5000 volumes at an average cost of £3, 16s. 6d. each.

4. Express decimally (correct to three places) the fourth part of £13, 17s. $4\frac{3}{4}$d.

Exact Results

Since 1d. $=£\frac{1}{240}=\dfrac{£4\frac{1}{6}}{1000}=£{\cdot}004\frac{1}{6}$, then, any number of pence may be expressed exactly as £'s by multiplying the pence by $4\frac{1}{6}$ and stating the result as thousandths of £1.

Examples. Express as exact decimals of £1 : (*a*) $5\frac{3}{4}$*d.*, (*b*) $8\frac{1}{2}$*d.*, (*c*) £3, 17*s.* $11\frac{1}{4}$*d.*

$$\begin{array}{lrl}
(a)\ 5\tfrac{3}{4}\text{d.} = & 5{\cdot}75\times 4 & =£{\cdot}023 \\
 & \textbf{plus}\ 5{\cdot}75\div 6 & =£{\cdot}000958\dot{3} \\
 & & =£{\cdot}023958\dot{3} \\
(b)\ 8\tfrac{1}{2}\text{d.} = & 8{\cdot}5\times 4 & =£{\cdot}034 \\
 & \textbf{plus}\ 8{\cdot}5\div 6 & =£{\cdot}00141\dot{6} \\
 & & £{\cdot}03541\dot{6} \\
(c)\ £3,\ 17\text{s. } 0\text{d.} & & =£3{\cdot}85 \\
\quad 11\tfrac{1}{4}\text{d.} = & 11{\cdot}25\times 4 & =£\ {\cdot}045 \\
 & \textbf{plus}\ 11{\cdot}25\div 6 & =£\ {\cdot}001875 \\
 & & £3{\cdot}896875
\end{array}$$

Exercise 10

1. Express as exact decimals of £1 :

(*a*) £3, 3s. $11\frac{1}{2}$d.; £4, 15s. $10\frac{3}{4}$d.; £1, 14s. $9\frac{1}{4}$d.; £2, 2s. $2\frac{1}{2}$d.

(*b*) 17s. $6\frac{1}{4}$d.; 19s. $8\frac{1}{2}$d.; 13s. $5\frac{1}{2}$d.; 12s. $4\frac{3}{4}$d.

2. Find, to the nearest farthing, the sum of ·844 of 10s.; 1·434 of 2s. 6d.; ·21 of 5s.

3. Find, to the nearest penny, the sum of

(*a*) ·8 of £3; $\frac{1}{2}$ of ·75 of 2s. 6d.; 2·2385 of 6s. 8d.; $\frac{1}{3}$ of £·963.

(*b*) $\frac{4}{7}$ of £·833; $\frac{5}{8}$ of £9·631; $\frac{2}{3}$ of £18·627.

4. If 2000 ozs. of gold are worth £9788, find exactly in £, s. d. the value per oz.

5. Express as an exact decimal of £1 the value of a fifth part of £13, 17s. $4\frac{3}{4}$d.

6. Find the cost of 1000 cwts. of tea at £11, 18s. 9d. per cwt.

7. Find the cost of 500 firkins of butter at £3, 13s. 9d. per firkin.

8. Find the cost of 20,000 sacks of flour at £2, 1s. 6d. per sack.

9. Find the cost of 400 tons of coal at 38s. 10½d. per ton.

10. Find the cost of 3000 acres of land at £20, 14s. 6d. per acre.

11. Find the cost of 1000 articles at (*a*) 2s. 6d. each; (*b*) 4s. 6d. each; (*c*) 12s. 6d. each.

12. Find the cost of 1000 articles at (*a*) 3s. 9d. each; (*b*) 7s. 9d. each; (*c*) 14s. 9d. each.

13. Find the cost of 1000 articles at (*a*) 3s. 3d. each; (*b*) 15s. 3d. each; (*c*) 19s. 3d. each.

14. Find the cost of 750 bicycles at £5, 10s. 6d. each.

15. Find the value of 4000 blankets at 14s. 10½d. each.

16. A town had a rateable value of half a million £'s. Find the amount received when the rate was 11s. 5½d. in the £.

17. Mr Kay bought 500 shares at 10s. each and sold them at 4s. 6½d. each. How much did he lose?

18. When the value of a dollar is 4s. 1½d., find the worth of 10,000 dollars in English money.

19. A merchant bought 1700 metres of silk at 7s. 9d. a metre and sold them at 9s. 1½d. per metre. Find his profit.

20. Find the interest on £30,000 for a year at £4, 15s. 6d. per cent. per annum.

Ratio

(i) The ratio of a to b is expressed thus, $a : b$.

(ii) The ratio of £5, 10s. 0d. to 15s. is $\frac{110}{15}$ or $\frac{22}{3}$. That is, £5, 10s. 0d. is $\frac{22}{3}$ times 15s. And the ratio of 5 tons 10 cwts. to 15 cwts, is similarly $\frac{110}{15}$ or $\frac{22}{3}$. Thus, we see that a ratio is a number and is independent of the **kind** of quantities compared.

(iii) Mutual ratios between more than two quantities of the same kind may be expressed. Thus, 5s. : 3s. 6d. : 2s. 6d. = 10 : 7 : 5.

Example. Divide £5, 15s. 0d. into three parts in the ratio 10 : 8 : 5.

The total number of units $= 10 + 8 + 5 = 23$.

∴ The first part $= \frac{10}{23}$ of £5, 15s. 0d. = £2, 10s. 0d.

The second part $= \frac{8}{23}$ of £5, 15s. 0d. = £2, 0s. 0d.

The third part $= \frac{5}{23}$ of £5, 15s. 0d. = £1, 5s. 0d.

Exercise 11

1. Fill in the blanks to make the second ratio equal to the first:

(*a*) $\frac{7}{14} = \frac{\ }{8}$; (*b*) $\frac{12}{8} = \frac{17}{\ }$; (*c*) $\frac{3}{7} = \frac{\frac{1}{3}}{\ }$;

(d) $\dfrac{70}{40}=\dfrac{\ }{3}$; (e) $\dfrac{1}{11}=\dfrac{\ }{3}$; (f) $\dfrac{9}{5}=\dfrac{\ }{\frac{1}{3}}$;

(g) $\dfrac{4}{9}=\dfrac{2^2}{\ }$; (h) $\dfrac{1}{r}=\dfrac{r}{\ }$; (i) $\dfrac{r}{1}=\dfrac{\ }{rt}$.

2. Mr J.'s salary is to Mr R.'s as 7 : 5. If Mr R.'s salary is £455, find Mr J.'s.

3. Express the following ratios in their simplest forms :

(a) $1\frac{1}{2} : 1\frac{2}{3}$; (b) $3\frac{1}{2} : 2\frac{1}{3}$; (c) $9\frac{3}{4} : 5\frac{4}{7}$;
(d) $4xy : 9y^2$; (e) $3b^2 : 15ab$; (f) $\frac{1}{4}x^2 : \frac{3}{8}xy$.

4. Find the multiplying factor which increases (a) 5 in the ratio 3 : 4 ; (b) £2 in the ratio 5 : 6 ; (c) £10 in the ratio $x : y$, and state the increased amount.

5. Increase (a) £50 in the ratio 2 : 3 ; (b) £120 in the ratio 3 : 11 ; (c) £200 in the ratio 5 : 9.

6. Find the multiplying factor which increases (a) 40 to 45 ; (b) 70 to 80 ; (c) 90 to 120.

7. Find the multiplying factor which decreases (a) 72 to 15 ; (b) 30 to 22 ; (c) 105 to 9.

8. Tom's marks in an examination were increased from 125 to 140. What is the increase per 100 ?

9. Ben's weekly wages were increased from 25s. to 27s. 6d. To what amount should Tom's wages of 22s. 6d. be raised to maintain the same ratio ?

10. Father's weight decreased from 12 stone 2 lbs. to 11 stone 6 lbs. If his son weighed 7 stone 4 lbs. and this decreased in the same ratio, find the son's new weight.

11. The population of two towns increased in the same ratio. The population of the first increased from 85,000 to 95,000. To what number did the second increase from 71,400 ?

12. Divide £100 among A, B, C, so that B may have three times as much as A, and C four times as much as B.

13. A, B, and C are partners in a business. The total capital is £20,000, of which A owns $\frac{1}{2}$, B $\frac{2}{5}$, and C the remainder. The profit for a year is £1400, and is divided in the ratio of the amount of capital owned. How much of the profit does each receive ?

14. Divide £55 into parts whose mutual ratios are $2\frac{1}{4} : 1\frac{3}{4} : 1\frac{1}{2}$.

15. Express in the simplest form : (a) $3\frac{1}{2} : 4\frac{1}{4} : 2\frac{1}{4}$; (b) $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$; (c) $5\frac{1}{2} : 7\frac{1}{2} : 9\frac{1}{2} : 10\frac{1}{2}$.

16. Mr Brown's salary is reduced by $\frac{1}{8}$. What fraction of

his new salary must be added to increase it to its original amount ?

17. Divide a line $4\frac{1}{2}$ ins. long in the ratio 2 : 3 : 4. What is the length of each part ?

18. A merchant lost $\frac{1}{7}$ of his stock and then found that the remainder was worth £20,300, 13s. 0d. What was the worth of the original stock ?

Exercises in proportion may be worked by means of ratios, thus :

Example. If $25\frac{1}{2}$ yds. of velvet cost £14, 13s. 3d., find the cost of $13\frac{1}{2}$ yds. of such velvet.

$$\frac{\text{The cost of } 13\frac{1}{2} \text{ yds.}}{\text{The cost of } 25\frac{1}{2} \text{ yds.}} = \frac{13\frac{1}{2}}{25\frac{1}{2}}$$

$$\begin{aligned}\therefore \text{ The cost of } 13\tfrac{1}{2} \text{ yds.} &= \frac{13\frac{1}{2}}{25\frac{1}{2}} \times £14,\ 13\text{s. } 3\text{d.}\\ &= \frac{27}{51} \times £14 \cdot 6625.\\ &= £7 \cdot 7625.\\ &= \underline{£7,\ 15\text{s. } 3\text{d.}}\end{aligned}$$

Exercise 12

1. When a train is timed to do a journey of $62\frac{3}{4}$ miles in 2 hrs., find the average speed in feet per second.

2. If 275 tons can be sent by railway a certain distance for £385, what will it cost, at the same rate, to send 195 tons the same distance ?

3. A tradesman is allowed $7\frac{1}{2}$ per cent. discount for cash, and so buys his goods for a net price of £510, 15s. 0d. What would he have had to pay if he had not been allowed discount ? Give your answer to the nearest shilling.

4. Two pipes, A and B, can fill a tank in $1\frac{1}{2}$ hrs. when running together. A can fill it in $2\frac{1}{4}$ hrs. when running alone. Both pipes are set running for 20 minutes and then A is stopped. In what time will B finish filling the tank ?

5. If lemons were bought at 5s. 6d. for 120 and sold at 6s. for 100, and a profit of £4, 5s. 0d. was made, how many were sold ?

6. By buying bananas at 36 for 2s. 6d. and selling them at

2s. 3d. a score, a trader gained £2, 6s. 6d. Find the number of bananas sold.

7. A cyclist travels 10 miles per hour, whilst a motorist travels 25 miles per hour. If the cyclist has a start of 18 miles, how many miles will the motorist have travelled when he overtakes the cyclist ?

8. When are the hands of a clock exactly together between (*a*) 3 and 4 o'clock, (*b*) 7 and 8 o'clock, (*c*) 9 and 10 o'clock ?

9. £385 was shared amongst A, B, and C, so that B had $\frac{3}{8}$ of A's share and C had as much as A and B together. How much did each receive ?

10. If 54 men can mend $\frac{7}{11}$ of a road in $2\frac{3}{4}$ days, how much longer will they take to complete the work if they continue at the same rate ?

Specific Gravity

The Specific Gravity (S.G.) of a substance is the ratio of the weight of the substance to the weight of an equal volume of water.

Example. Find the S.G. of pure lead, having given that a sample of pure lead weighs 22·72 grams and an equal volume of water weighs 2 grams.

$$\text{S.G.} = \frac{\text{Weight of lead}}{\text{Weight of an equal volume of water}},$$

$$= \frac{22{\cdot}72 \text{ grams}}{2 \text{ grams}} = \underline{11{\cdot}36.}$$

Exercise 13

1. What is the weight of a square decimetre of sheet iron ·3 cm. in thickness if the S.G. of iron is 7·2 ?

2. Given that the S.G. of rock crystal is 2·65, find the weight of (i) a cube of this substance having an edge 3 cm. in length ; (ii) a prism of the substance 4 cm. × 3 cm. × 2·4 cm.

3. A cubic foot of water weighs 62·3 lbs. and a cubic foot of zinc weighs 428 lbs. Find (i) the S.G. of zinc, (ii) the weight in lbs. of a tray made up from a sheet of this metal 6 ft. × $2\frac{1}{2}$ ft. × $\frac{1}{2}$ in. when $\frac{1}{16}$ cu. ft. is wasted in the manufacture. (Answer correct to two places of decimals.)

4. The S.G. of turpentine is ·88. Find the weight (in lbs.) of water contained in a vessel which, if filled to the same depth with turpentine, would contain 55·35 lbs. of liquid.

5. A plate of iron 290 cm. $\times$ 185 cm. $\times$ 22 mm. weighs 849·8 Kg. Find (i) the S.G. of this iron, (ii) the volume of such iron which would counterbalance a kilogram of water.

6. A rectangular pedestal is 5 ft. $\times 6\frac{1}{2}$ ft. $\times 4\frac{3}{4}$ ft. If the S.G. of the stone forming the pedestal is 2·4, find its weight in tons (1 cu. ft. of water weighs 62·3 lbs.).

7. A tank, 25 decimetres long and 1·6 metre broad, contains 1080 Kg. of mercury. If the S.G. of mercury is 13·5, find the depth of the liquid.

8. A tank 3 ft. $\times 2\frac{1}{2}$ ft. $\times 4\frac{1}{4}$ ft. holds 1985·8 lbs. when filled with fresh water, and 2049·5 lbs. when filled with sea water. Find the S.G. of sea water.

Proportion

When considering ratio we compare one quantity with another; when considering proportion we make a statement of the equality of ratios.

$$\text{The value of the ratio } 8:2=\tfrac{8}{2}=4.$$
$$\quad ,, \qquad ,, \qquad ,, \quad 12:3=\tfrac{12}{3}=4.$$
$$\therefore\ 8:2=12:3 \quad \text{or} \quad \tfrac{8}{2}=\tfrac{12}{3}.$$

This equality of ratios forms a proportion and may be expressed thus: 8 : 2 : : 12 : 3, and is read: 8 is to 2 as 12 is to 3.

Four quantities are said to be in proportion when the relation which the first bears to the second is the same as that which the third bears to the fourth, *e.g.*

$$21:14::36:24, \text{ for } \tfrac{21}{14}=\tfrac{36}{24} \text{ and both }=\tfrac{3}{2}.$$
$$\text{and } 35:15::21:9, \text{ for } \tfrac{35}{15}=\tfrac{21}{9} \text{ and both }=\tfrac{7}{3}.$$

The first and fourth of the four terms are known as the **extremes,** and the second and third are known as the **means.**

1st term		2nd term		3rd term		4th term
21	:	14	: :	36	:	24

Means

Extremes

When four terms are in proportion, as above, the product of the means = the product of the extremes. In the last example $21 \times 24 = 14 \times 36 = 504$.

We may show it generally thus:

$$\text{Let } \frac{a}{b}=\frac{c}{d}.$$

Multiply, as before, by the product of the denominators.

Then $$\frac{a \times \cancel{b} \times d}{\cancel{b}} = \frac{c \times b \times \cancel{d}}{\cancel{d}}.$$

$$\therefore \; ad = cb.$$

That is, the product of the means = the product of the extremes.

From this statement we may find any one of the missing terms of a proportion.

Example. Find the missing term in the proportion :

$$x : 64 :: 36 : 512.$$

$$\therefore \; 512x = 36 \times 64$$

$$\therefore \quad x = \frac{\overset{9}{\cancel{36}} \times \cancel{64}}{\cancel{512}\;\cancel{128}\;2}$$

$$= 4\tfrac{1}{2}.$$

Check : $4\frac{1}{2} \times 512 = 36 \times 64 = 2304$.

Exercise 14

1. What number bears to 65 the same ratio that $4\frac{1}{2}$ bears to $6\frac{1}{2}$?

2. Find the fourth proportional to : (i) 8, 17, $34\frac{2}{3}$; (ii) 2·8, 23, ·315 ; (iii) $\frac{2}{3}$, $\frac{7}{16}$, $\frac{4}{10}$; (iv) 56, 460, 630.

3. Fill in the missing items in the following proportions :

(i) £8 : £5, 10s. 0d. :: ? : 28 tons.

(ii) $2\frac{1}{4}$ lbs. : ? :: 9s. : 13s. 9d.

(iii) 8 lbs. : ? :: ? : 18 lbs.

(iv) 1 ft. : ? :: 5·8 m. : ·58 Km.

4. Seven men and 3 boys finish a piece of work in 18 days. How long will it take 9 men and 14 boys to do the work if one man's work is equal to the work of 3 boys ?

5. The ratio of a metre to a yard is 219 : 200. Using this fact, find the distance to the nearest yard equivalent to 3960 m.

6. The tax on certain goods is assessed at $6\frac{3}{4}$d. in the £. Find the tax due on goods worth £57, 12s. 6d.

7. The extension of a spring is directly proportional to the force applied. When a spring stretches 2·5 cm. by the application of a force of 5 Kg., what force must be applied to stretch the spring 3·5 cm. ?

8. A wheel having a diameter of 3 ft. 6 ins. travels $1\frac{3}{4}$ miles. How often does it revolve ? Find by proportion how often

a wheel with a diameter 5 ft. 3 ins. would revolve in travelling the same distance. ($\pi = \frac{22}{7}$.)

9. When 1 ton of cabbages cost £2·25, find the cost of 5·05 cwts. Give the answer to the nearest penny.

10. A snail starts in the morning of 1st August to climb a wall 10 ft. high at the rate of $1\frac{1}{2}$ ft. per day, but during each night it slips down 5 ins. On what date will it first reach the top of the wall?

EXERCISE 15

Say why the following questions cannot be answered:

1. A man who takes a preliminary run of 10 yds. can jump 15 ft.; how long will be his preliminary run in order to jump 100 yds.?

2. The first four chapters of a book are printed on 52 pages; how many pages will be required for eight chapters?

3. A boy 5 ft. in height weighs 7 stone 6 lbs.; how tall will he be when he weighs 11 stone?

4. A fisherman lands 10 fish in 20 mins.; how many will he land in an hour?

5. A girl skips 35 times in half a minute; how long will she take to skip 1000 times?

6. A boy spends 10s. during a month's holiday; how long will it take him to spend £100?

Compound Proportion

The equality of the ratio of two quantities to the ratio of two other quantities is known as **Simple Proportion**; but when a third ratio is introduced we speak of the equality as **Compound Proportion.**

Example. If 32 *men can build a wall* 96 *yds. long in* 4 *days, in what time could* 18 *men build a wall* 540 *yds. long, working at the same rate?*

We reason thus:

(i) If the length of the wall remain constant (say 96 yds.), and the number of men is decreased to 18, then the time will be increased in the ratio 18 : 32, that is: Time $= 4 \times \frac{32}{18}$ days.

(ii) If the length of the wall be now increased from 96 yds. to 540 yds., the time will again be increased in the ratio 96 : 540.

$$\therefore \text{ the time required} = 4 \times \tfrac{32}{18} \times \tfrac{540}{96}.$$
$$= 40 \text{ days.}$$

Graphical Illustration.—A rectangle ABCD (fig. 2) has an area of (15 × 10) squares. When the length AB is increased in the

ratio 3: 5 and the altitude AD is increased in the ratio 2 : 3, what is the area of the new rectangle ?

(i) When AB is increased in the ratio 3 : 5 it becomes AE, and the first new rectangle becomes AEFD, which $= \frac{5}{3}$ of ABCD.

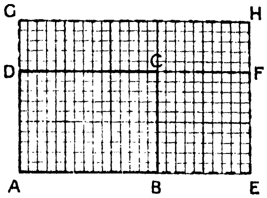


Fig. 2.

(ii) When AD is increased in the ratio 2 : 3 it becomes AG, and the final new rectangle becomes AEHG, $= \frac{3}{2}$ of AEFD, $= \frac{3}{2} \times \frac{5}{3}$ of ABCD, $= 2\frac{1}{2}$ times ABCD.

Check : Rectangle ABCD contains 150 squares.
Rectangle AEHG contains 375 squares, *i.e.* $2\frac{1}{2}$ times 150 squares.

Exercise 16

1. Ninety-eight men working 9 hrs. per day can pump a reservoir dry in 66 days. If the number of men were increased by 100 and the working day decreased by 3 hrs., how long should the work last ?

2. Three men earn £18 in 20 days. How many men working at the same rate of pay can earn 30 guineas in 15 days ?

3. In making a book of 96 pages 132 reams of paper are used for 1200 copies. If twice as many copies were issued, and the number of pages decreased by 24, how many reams would then be required ?

4. If 6 men can do as much work as 7 women or as 8 boys, how many boys must help 30 men and 20 women to do in 6 days the work which it takes 100 boys 8 days to do ?

5. A 5-lb. loaf costs 1s. $1\frac{1}{2}$d. when wheat is £5, 2s. 6d. per quarter. What is the price of wheat when a 2-lb. loaf costs $5\frac{1}{2}$d. ?

6. How far should 70 cwts. be carried for £27, 10s. 0d. if 30 cwts. are carried 17 miles for £6, 5s. 9d. ?

7. A regiment of 1700 men has provisions for 14 weeks. More troops join the regiment and no extra provisions are allowed. If each man's ration is reduced in the ratio 6 : 5, the provisions will last 10 weeks. How many men join the regiment ?

8. A steam turbine works at 4800 horse-power and consumes 450 tons of coal in a week. Find (in lbs.) the consumption per hour per horse-power.

9. In a workshop 500 men were employed, the working hours per week being 44, and the weekly wages amounted to £1337, 10s. 0d. At a later period 100 extra men were employed, and all the men worked overtime for 2 hrs. per day on 5 days of the week, this being paid for at $1\frac{1}{2}$ times the usual rate. If all the men were paid alike, calculate the total weekly wages during this later period.

10. If 756 Kg. can be carried by rail a distance of 350 Km. for 900 francs, for what distance can 850 Kg. be carried for 750 francs? (Give the answer to the nearest Km.)

Also find, to the nearest franc, the corresponding charge for the carriage of 1000 Kg. a distance of 1000 Km.

Percentage

When two quantities are expressed as a ratio, *e.g.* 4 : 5 or $\frac{4}{5}$, we may transform the fraction so that the denominator is 100. The ratio is then said to be expressed as a percentage. Thus $\frac{4}{5}=\frac{80}{100}$, and so we say that $\frac{4}{5}$ is the same as 80 per cent.

Example I. Express $\frac{7}{32}$ as a percentage.

Let x = rate per cent.

Then
$$\frac{x}{100}=\frac{7}{32}.$$

$$\therefore\ x=\tfrac{7}{32}\times 100=\tfrac{700}{32}=21\tfrac{7}{8}\quad\therefore\ \text{Rate}=21\tfrac{7}{8}\text{ per cent.}$$

Example II. What percentage of $5\frac{1}{2}$ tons is 7 cwts.?

$$\frac{7\text{ cwts.}}{5\frac{1}{2}\text{ tons}}=\frac{7}{110}=\left(\frac{7}{110}\times 100\right)\text{ per cent.}=6\tfrac{4}{11}\text{ per cent.}$$

Example III. Of what number is 18 = 3 per cent.?

Let x = the number.

Then
$$\frac{3x}{100}=18.$$
$$\therefore\ x=\tfrac{1800}{3}=600.$$

Commission is an allowance paid to an agent for carrying out a business transaction. It is generally calculated on the amount of money spent.

Brokerage is the commission charged by a broker.

A premium is a sum paid periodically to an office for insurance, *e.g.* against fire or loss of life or property.

Exercise 17

1. Find (correct to 3 places) the percentages equivalent to: (i) $\frac{3}{8}$, (ii) $\frac{7}{15}$, (iii) $\frac{9}{16}$, (iv) $\frac{11}{23}$, (v) $\frac{5}{14}$, (vi) $\frac{9}{32}$.

2. Find the fractions equivalent to:

(i) 3·5 per cent., (ii) $12\frac{1}{8}$ per cent., (iii) $33\frac{1}{3}$ per cent., (iv) $4\frac{1}{2}$ per cent., (v) ·1 per cent., (vi) $63\frac{2}{3}$ per cent.

3. What is the sum of which 63 per cent. amounts to £27, 15s. 6d. ?

4. What percentage is £61, 12s. 0d. of £1155 ?

5. An agent is paid $2\frac{1}{4}$ per cent. commission for collecting rents. If the rents amount to £582, how much commission does he receive ?

6. In measuring a furlong the length is measured 4 ins. too long. Find the error as a percentage of the correct length.

7. In 1900 the population of a town was 8590. In 1925 the population was 9465. Find the increase as a percentage of the population in 1900.

8. A regiment lost $\frac{2}{7}$ of its troops as casualties and 5 per cent. of the remainder through sickness. If 12,540 men were then left, what was the original strength of the regiment ?

9. A man buys mill shares to the value of £8271. If he pays $3\frac{1}{3}$ per cent. for brokerage and £5 for stamp duty, how much does he pay altogether ?

10. An insurance company charges $8\frac{1}{2}$ per cent. as premium against loss of cargo. How much does it cost to insure a cargo worth £9150 ? How much must be paid to secure the return of the premium also ?

11. After deducting a brokerage of $2\frac{1}{2}$ per cent., a bill amounts to £647, 3s. 10d. What was the original amount ?

12. Mr R. lost 15 per cent. when he sold his house for £790, 10s. 0d. What did the house cost him ?

13. x per cent. of a number is $12\frac{1}{2}$. What is the number ?

14. Find, to the nearest penny, the net amount payable after deducting successively,

(i) 20 per cent., $2\frac{1}{2}$ per cent., from £55, 15s. 6d.

(ii) 15 per cent., 5 per cent., 5 per cent., from £105, 15s. 0d.

(iii) 20 per cent., $7\frac{1}{2}$ per cent., $2\frac{1}{2}$ per cent., from £2020, 10s. 0d.

15. Find the selling price of goods bought for £18, 14s. 0d. and sold at a gross profit of 15 per cent. on the *selling* price.

16. How much per cent. above the cost price must a tradesman mark his goods so that after allowing 5 per cent. discount on the marked price he may gain 15 per cent. on the cost price ?

17. A tea dealer sells tea at 2s. 6d. per lb., thereby clearing 25 per cent. on his outlay. The duty on tea being reduced by

3d. per lb., he lowers his selling price 3d. per lb. What is now his profit per cent. on the C.P. ?

18. A bought goods for £65, 12s. 6d. and sold them to B at a profit of 12 per cent. on his C.P. B sold the goods to C for £79, 0s. 3d. What profit per cent. on the C.P. did B make ?

19. A grocer bought 5 cwts. 1 qr. 22 lbs. of soap for £22, 16s. 3d. He sold $2\frac{1}{2}$ cwts. at $10\frac{1}{2}$d. per lb. and the remainder at 11d. per lb. What profit per cent. did he make on his outlay ?

20. A partner in a business worth £19,200 sells $\frac{5}{16}$ of his share for £2500. What percentage of the business does he still own ?

Simple Interest

Work exercises in Simple Interest by decimals and so simplify and shorten calculations.

Remember that it is generally sufficient to give the result correct to the nearest penny.

Example. Find the S.I. on £872, 17s. 6d. for 9 months at $3\frac{1}{2}$ per cent. per annum.

As $$\text{S.I.} = £\frac{P \times R \times T}{100}.$$

In this case $$\text{S.I.} = £\frac{872{\cdot}875 \times 3 \times 7}{4 \times 2 \times 100}$$

$= £22{\cdot}913$ (correct to 3 places).

$= £22$, 18s. 3d. (correct to the nearest penny).

Short Methods

I. Since 10 per cent. of £1 $= £\frac{1}{10} = 2$s., 5 per cent. of £1 $= £\frac{1}{20} = 1$s., $2\frac{1}{2}$ per cent. of £1 $= £\frac{1}{40} = 6$d., then

S.I. on £1 at 10 per cent. per annum = 2s.
,, ,, 5 ,, ,, ,, = 1s.
,, ,, $2\frac{1}{2}$,, ,, ,, = 6d.

Therefore the S.I. on £68 for a year at 5 per cent. = 68s. = £3, 8s. 0d., at 10 per cent. $= 68 \times 2$s. $= £6$, 16s. 0d. : at $2\frac{1}{2}$ per cent. $= 68 \times 6$d. $= £1$, 14s. 0d.

II. Consider this example:

Find the S.I. on £40 for 1 year at 3 per cent. per annum.

$$\text{S.I.} = \frac{£40 \times 3}{100} = \frac{40 \times 3 \times 20}{100}\,\text{sh.} = \frac{40 \times 3 \times 2}{10}\,\text{sh.} = 24\text{s.} = £1,\ 4\text{s.}\ 0\text{d.}$$

Note that when the result is expressed in shillings it is equivalent

to $\left(\frac{P \times R \times 2}{10}\right)$ shillings. Apply this formula to find mentally the S.I. for one year on £80 at $2\frac{1}{2}$ per cent., $3\frac{1}{2}$ per cent., 4 per cent., and so on.

EXERCISE 18

1. Find mentally the S.I. on :

(*a*) £70 at 2 per cent. per annum for 1 year.
(*b*) £90 at $2\frac{1}{2}$ per cent. per annum for 2 years.
(*c*) £35 at 5 per cent. per annum for 3 years.
(*d*) £20 at 7 per cent. per annum for 1 year.
(*e*) £45 at 5 per cent. per annum for 2 years.

2. Find, to the nearest penny, the S.I. on :

(*a*) £870 for 148 days at $3\frac{1}{4}$ per cent. per annum.
(*b*) £640, 8s. 4d. for 2 years 219 days at $1\frac{1}{4}$ per cent. per annum.
(*c*) £475, 12s. 0d. from 1st March to 25th July at $3\frac{1}{2}$ per cent. per annum.
(*d*) £603, 10s. 9d. from 16th April to 2nd August at $4\frac{1}{8}$ per cent. per annum.
(*e*) £216, 13s. 8d. for 146 days at 2·8 per cent. per annum.
(*f*) £1808 for 1 year 35 days at $4\frac{3}{4}$ per cent. per annum.
(*g*) £802, 16s. 3d. from 3rd March to 3rd May at $1\frac{5}{8}$ per cent. per annum.

3. A certain corporation pays interest half-yearly at the rate of $6\frac{1}{4}$ per cent. per annum. What will be the total simple interest paid on a loan of £475 in a period of 6 years ?

4. If accounts are not paid on the day they are due a manufacturer charges interest on the amount at the rate of $5\frac{1}{2}$ per cent. per annum. If a bill for £2870 was due on 6th April, and was not settled till 3rd May, find, to the nearest penny, the amount then paid.

5. On 1st January 1926 a man invested £2000 at $4\frac{1}{2}$ per cent. per annum. After 5 months he withdrew £670. What could he draw as interest on 1st January 1927 ?

6. Find the amount (Principal + S.I.) of :

(*a*) £670 after 8 months at $4\frac{1}{2}$ per cent. per annum.
(*b*) £840, 10s. 0d. after 73 days at $6\frac{1}{4}$ per cent. per annum.
(*c*) £83, 18s. 6d. after 1 year 146 days at 8 per cent. per annum.

(*d*) £15, 7s. 4d. after 62 days at $4\frac{3}{4}$ per cent. per annum.

(*e*) £3921 invested on 3rd July and withdrawn on 18th September at 4 per cent. per annum.

(*f*) £762 borrowed on 4th August and repaid 14th November at $33\frac{1}{3}$ per cent. per annum.

7. Find the amount paid for an account of £840 due on 17th March and not paid till 3rd June, if interest on overdue account is 6 per cent. per annum.

8. Work as shortly as possible:

(*a*) S.I. on £100, 13s. 4d. for 6 years at 4 per cent. per annum.

(*b*) S.I. on £300 for 2 years 73 days at 5 per cent. per annum.

(*c*) S.I. on £620 for $1\frac{1}{2}$ years at 2 per cent. per annum.

9. What does a man gain yearly on an investment of £1506, 2s. 6d. if the firm with which the money is invested increases its rate from $4\frac{3}{4}$ per cent. per annum to $5\frac{1}{2}$ per cent. per annum?

10. A and B each wish to borrow £208 for 6 months. A borrows at the rate of $7\frac{1}{2}$ per cent. per annum, and B borrows at the rate of 1 shilling per pound per month. By how much does B's interest exceed A's?

To find the total S.I. on two or more different principals for different times, **when the rate of interest is the same,** we may use the formulæ

$$\text{S.I.} = \frac{\text{R}}{100} \times \text{P}_1\text{T}_1 \text{ and S.I.} = \frac{\text{R}}{100} \times \text{P}_2\text{T}_2.$$

Combining them, we obtain, Total S.I. $= \frac{\text{R}}{100}(\text{P}_1\text{T}_1 + \text{P}_2\text{T}_2)$.

Example. Find in one operation the total S.I. on (i) £840 *for 2 years, and on* (ii) £210 *for 3 years, when the interest in each case is at $4\frac{1}{2}$ per cent. per annum.*

$\text{P}_1 = £840, \text{T}_1 = 2$ years. $\text{P}_2 = £210, \text{T}_2 = 3$ years. $\text{R} = 4\frac{1}{2}$ per cent.

$$\begin{aligned} \therefore \text{ S.I.} &= £\frac{4\frac{1}{2}}{100}(840 \times 2 + 210 \times 3). \\ &= £\frac{9}{200}(1680 + 630). \\ &= £\frac{9}{200} \times 2310. \\ &= £103,\ 19\text{s. } 0\text{d.} \end{aligned}$$

EXERCISE 19

1. A man borrows £7326 from a bank at $4\frac{1}{4}$ per cent. per annum, and after 2 months the rate is raised to $4\frac{3}{4}$ per cent. If the debt is settled 8 months from the date on which the money was borrowed, find the total interest to be paid.

2. Find, to the nearest penny, the total S.I. at $3\frac{3}{4}$ per cent. per annum on £418, 10s. 0d. invested for 8 months, £39 for 4 months, and £136, 0s. 0d. for 9 months.

3. Find, to the nearest penny, the total S.I. for 73 days on £357 invested at $2\frac{1}{2}$ per cent. per annum, £431, 10s. 0d. at 3 per cent. per annum, and £17, 18s. 0d. at $4\frac{1}{2}$ per cent. per annum.

4. At the beginning of a year a man invested £2000 at 3 per cent. per annum. After 5 months he withdrew the interest and £452. After 9 months he withdrew £326 and the interest. If the rest remained to the end of the year, what was the total S.I. the man received ?

5. A merchant taking a trip for 10 months leaves behind him £650 invested at $4\frac{1}{2}$ per cent. per annum, £974 invested at $3\frac{3}{4}$ per cent. per annum, and £325 invested at $4\frac{3}{4}$ per cent. per annum. What amount of S.I. will he be able to claim on his return ?

6. A man gave each of his three daughters £480. The first invested her money for 8 months at 5 per cent., the second for 11 months at $4\frac{1}{2}$ per cent., and the third for 10 months at $4\frac{3}{4}$ per cent. What was the total S.I. received by the daughters ?

7. On 1st January 1926 a man invests £29, 10s. 0d. with a firm paying 5 per cent. per annum, and £38, 15s. 0d. with a firm paying 4 per cent. per annum. If he closes both accounts on 6th July 1926, how much money should he withdraw altogether ?

8. Find, to the nearest penny, the total S.I., at $3\frac{3}{4}$ per cent. per annum, on £513 lent for 3 months, £65 for 73 days, and £87 for 146 days.

9. If one Corporation pays $4\frac{1}{2}$ per cent. per annum on loan money, whilst another pays $4\frac{3}{4}$ per cent. per annum, how much S.I. would be received if £90 were invested in the first and £120 in the second, both amounts for 18 months ?

10. At the beginning of a year a man borrows £4800 from a bank where the rate charged is $3\frac{1}{4}$ per cent. After 4 months

the rate rises to $4\frac{1}{2}$ per cent. per annum, and 3 months later to 5 per cent. per annum. If the rate continues at 5 per cent. for the rest of the year, how much S.I. will the man have to pay at the end of the year?

Compound Interest

When the interest on becoming due is added to the previous principal, a new principal is formed, and interest is then calculated on the new principal. This process is known as Compound Interest (C.I.).

Example. Find, to the nearest penny, the C.I. on £785 for 2 years at $4\frac{1}{2}$ per cent. per annum.

	£	
	785	P. for 1st year.
4 per cent. . .	31·4	
$\frac{1}{2}$,, . .	3·925	
	820·325	P. for 2nd year.
4 ,, . .	32·813	
$\frac{1}{2}$,, . .	4·1016	
	857·2396	Amount at end of 2nd year.
	785	Original P.
	72·2396	C.I. for 2 years.
	= £72, 4s. 10d	

Exercise 20

(Give answers to the nearest penny.)

1. Find the C.I. on:

(*a*) £400 for 2 years at 4 per cent. per annum.
(*b*) £750 for 3 years at 5 per cent. per annum.
(*c*) £1000 for 3 years at $2\frac{1}{2}$ per cent. per annum.
(*d*) £200 for 3 years at $4\frac{1}{2}$ per cent. per annum.

2. Find the amount of £720 for 2 years at 3 per cent. per annum.

3. A mortgage of £500, rate of interest 6 per cent., was neglected for 3 years. What was the total amount then due, C.I. being charged?

4. Find the increase on £120 put out for two years at 5 per cent. per annum C.I., the interest being added half-yearly.

5. A man borrowed £200 at $4\frac{1}{2}$ per cent. C.I. At the end

of each year for two years he paid £50. How much had he to pay at the end of the third year in order to settle the debt ?

6. How much will £100 amount to if left to accumulate for 3 years at 5 per cent. per annum C.I. ?

7. If I invested £50 in a bank at 4 per cent. per annum, how much should I be able to withdraw at the end of 4 years, if the interest due were credited to my account yearly ?

8. A sum of £250 was left to accumulate for 5 years. What did it amount to at the end of that time, C.I. being reckoned at 2 per cent. per annum ?

9. Find the amount of £1000 in 3 years at 5 per cent. per annum C.I.

10. The working plant of a firm was originally worth £15,000. It is depreciated by 5 per cent. at the end of each year. What will be its estimated value at the beginning of the fifth year ?

Money—British and Foreign

British Money

When a boy exchanges a number of marbles for a penknife, the transaction is spoken of as **barter.** This method of trading by exchanging actual goods is the oldest method known. But it is often difficult to barter goods because of the difference in their values. To overcome the difficulties connected with barter the values of articles of commerce are now generally estimated by the amount of money they are worth, and money is said to be the **common medium of exchange.** By this means goods are bought and sold ; thus, a person having corn to sell can exchange it for a certain amount of money, and then with the money he can buy other commodities he may require.

Money has two qualities: viz. (*a*) it is a standard of value; (*b*) it is an instrument of exchange.

British money includes, I. *Coins*, II. *Bank Notes and Treasury Notes*, III. *Postal and Money Orders*, IV. *Cheques and Bills of Exchange.*

I. *Coins.*—A new sovereign weighs 123·27447 grains, and of this $\frac{11}{12}$ is pure gold and $\frac{1}{12}$ is an alloy. Gold is said to be of as many carats as it contains twenty-fourth parts of pure metal. Therefore we say a sovereign is made of 22-carat gold. Fifteen-carat gold means gold containing $\frac{15}{24}$ of pure gold, and so on.

British **gold coins** are worth their face value as metal, and are known as **standard coins.**

Silver coins are not worth their face value as metal and are said to be **token coins.** The alloy used in minting silver coins

was established at 50 per cent. pure silver by the Coinage Act of 1920.

Bronze coins are **token coins** also, as they are minted from an alloy of 95 per cent. copper, 4 per cent. tin, and 1 per cent. zinc.

Troy Weight

(For weighing gold and silver.)

24 grains = 1 pennyweight (1 dwt.).
20 dwts. = 1 ounce (oz.).
12 ozs. = 1 pound (lb.).
5760 grains = 1 pound.

Note.—A grain Troy = a grain Avoirdupois, but a pound Troy contains 5760 grains, and a pound Avoirdupois 7000 grains. Therefore 1 lb. Troy : 1 lb. Avoirdupois = 5760 : 7000 = $\frac{14}{17}$ nearly.

II. *Bank Notes* of various values from £5 to £1000 are issued by the Bank of England, and *Treasury Notes* for 10s. and £1 by the Lords Commissioners of His Majesty's Treasury.

III. *Postal Orders* and *Money Orders* are issued by the Postal Authorities for the convenience of persons wishing to send small sums of money by post.

IV. *Cheques* may be issued by persons who have Current Accounts with a banker. A cheque is a particular form of a *Bill of Exchange* (see p. 32).

Foreign Money

Every foreign country of importance has its own currency and issues gold coins. In England 40 pounds (troy) of $\frac{11}{12}$ fineness (*i.e.* 22 carat) is minted into 1869 sovereigns. In France 1 Kg. of gold of $\frac{9}{10}$ fineness is coined into 155 twenty-franc pieces. By calculation it will be found that £1 is equal in value to 25·225 francs. This is the true value of £1 expressed in francs, and is called the **Mint par of Exchange**.

The Mint par of exchange can also be calculated in American dollars, in German marks, in Dutch florins, in Scandinavian kroner, etc.

The value of £1 in foreign coinage is always varying, the amount of variation depending very largely on the credit of the country concerned.

The daily newspapers publish lists showing these variations from day to day. For instance, the rates of exchange at a certain date were:

	Mint par value of £1.	Value of £1 at this date.
New York . .	$4{\cdot}86\frac{2}{3}$ dollars.	$4{\cdot}85\frac{11}{16}$ dollars.
Paris . . .	$25{\cdot}22\frac{1}{2}$ francs.	103·40 francs.
Berlin . . .	20·43 marks.	$20{\cdot}40\frac{1}{2}$ marks.
Amsterdam . .	12·107 florins.	$12{\cdot}05\frac{1}{8}$ florins.
Oslo . . .	18·159 kroner.	25·72 kroner.
Brussels . .	$25{\cdot}22\frac{1}{2}$ francs.	107·05 francs.
Copenhagen . .	18·159 kroner.	20·72 kroner.
Geneva . .	$25{\cdot}22\frac{1}{2}$ francs.	25·05 francs.
Bucharest . .	$25{\cdot}22\frac{1}{2}$ lei.	950 lei.
Athens . . .	$25{\cdot}22\frac{1}{2}$ drachmai.	317 drachmai.
Madrid . . .	$25{\cdot}22\frac{1}{2}$ pesetas.	$33{\cdot}71\frac{1}{2}$ pesetas.
Milan . . .	$25{\cdot}22\frac{1}{2}$ lire.	$133\frac{3}{4}$ lire.

The coinage of most foreign countries is based on the decimal system. Thus:

Country.	Coins.
United States .	100 cents = 1 dollar.
France . . .	100 centimes = 1 franc.
Belgium . .	100 centimes = 1 franc.
Switzerland . .	100 centimes = 1 franc.
Norway . .	100 ore = 1 krone.
Sweden . .	100 ore = 1 krone.
Denmark . .	100 ore = 1 krone.
Germany . .	100 pfennige = 1 mark.
Holland . .	100 cents = 1 florin or guilder.
Spain . . .	100 centesimos = 1 peseta.
Italy . . .	100 centesimi = 1 lira.
Brazil . . .	1000 reis = 1 milreis.

Exercise 21

(Work the problems to two decimal places.)

1. Given that a sovereign weighs 123·27447 grains, find the weight in lbs., dwts., grains (troy) of £5000 in sovereigns.

2. The British sovereign is 22-carat gold. Calculate the weight of pure gold contained in 3000 grains of standard coin. (Give the answer in grains.)

3. A banker exchanges 24·25 dollars for £5. Find, to the nearest penny, the value of 860 dollars.

4. When the rate of exchange between London and Paris is £1 = 38·17 francs, find the cost in francs of an article worth £88, 15s. 9d.

5. A merchant buys 20 Kg. of butter in Germany for 48·48 marks. What price is he paying per lb. if 1 Kg. = 2·2 lbs., and $25{\cdot}08\frac{2}{5}$ marks = £1 ?

6. If $\frac{11}{12}$ of English gold coinage is pure gold, and $\frac{9}{10}$ of French gold coinage is pure gold, find the Mint par of exchange between London and Paris if a sovereign weighs 123·27 grains and a twenty-franc piece weighs 99·56 grains.

7. If the price of French varnish is 184 Kg. for 327 francs in Paris, what is an English colour merchant's price per lb. if he makes 10 per cent. profit ? (Assume 1 franc = $8\frac{3}{4}$d. and 1 Kg. = $2\frac{1}{5}$ lbs.)

8. If pure gold is sold at £4, 5s. 0d. per ounce (troy), and 80 lbs. of 22-carat gold make 3738 sovereigns, find the value of the gold contained in nine sovereigns.

9. A merchant changes £982 into Dutch kroner at the rate of 18·2 kroner per £1. He visits Copenhagen, spends half his money, and on visiting Berlin converts the rest to marks. If 1 kroner = 1·12 marks, what is the German equivalent of the money he can spend in Berlin ?

10. A dealer converts £457 to dollars, thence to francs, and thence to kroner. How many kroner will he receive if the following rates of exchange operate : London to New York, £1 = 4·85 dollars ; New York to Paris, 1 dollar = 22·5 francs ; Paris to Stockholm, 1 krone = 6·55 francs ?

11. Find the value, to the nearest halfpenny, of one troy ounce (i) of standard gold, (ii) of pure gold when 20 troy pounds of standard gold are coined into $934\frac{1}{2}$ sovereigns.

12. Find the equivalents, to two decimal places, or as £, s. d., to the nearest penny, of the following at Mint par value :

(*a*) £50 in francs ;	(*e*) 1000 lire in £, s. d.
(*b*) £750 in dollars ;	(*f*) 100 dollars in £, s. d.
(*c*) £150 in marks ;	(*g*) 2000 francs in £, s. d.
(*d*) £200 in pesetas ;	(*h*) 500 kroner in £, s. d.

13. Find the cost of 85·75 m. of silk at 10 francs 50 centimes per metre.

14. On arriving at Chicago an English passenger changes £100 into dollars. How many will he receive at Mint par value ?

15. How much must a cotton merchant pay for a draft for

\$50,000 which he wishes to send to New Orleans, if the rate of exchange is $\$4\cdot86\frac{2}{3}$ (\$ = dollar, as £ = pound sterling)?

16. Find the cost of 500 Km. 50 m. of wire at 5 francs 75 centimes per Km.

17. Sixteen annas make a rupee. If the value of the rupee in English money is 1s. $5\frac{2}{32}$d., find the number of rupees and annas that should be obtained for £120. (Give answer to the nearest anna.)

18. Find, to the nearest cent, the value of £40, 15s. 6d. in Dutch florins and cents at Mint par value.

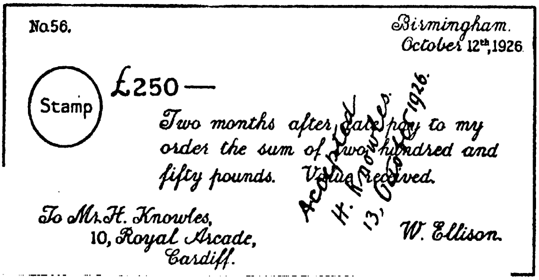
19. If £1 = 25·60 Swedish kroner, and a Spanish peseta is worth $9\frac{1}{2}$d., find the value in pesetas and centesimos of 500 kroner.

20. Find the value in marks of 45 Km. of wire at £12, 12s. 0d. per mile (1 mile = 1·6 Km.; £1 = 20·4 marks).

Exchange and Banker's (Commercial) Discount

A Bill of Exchange (B/E) is "an unconditional order in writing, addressed by one person to another, signed by the person giving it, requiring the person to whom it is addressed to pay on demand, or at a fixed or determinable future time, a certain sum in money to, or to the order of, a specified person, or to a bearer." There are two kinds of Bills of Exchange: (1) Inland, (2) Foreign. Inland Bills are payable in the British Isles, and Foreign Bills may be made payable almost anywhere. The advantages of a Bill of Exchange are: (1) it fixes the amount of the debt and it may be used like a cheque; (2) it prevents the carrying of coins to and from foreign countries; (3) the merchant shippers secure immediate payment.

The following is an example of an **Inland Bill of Exchange:**


No. 56.

Birmingham.
October 12th, 1926.

Stamp

£250—

Two months after date pay to my order the sum of Two hundred and fifty pounds. Value Received.

Accepted
H. Knowles
13 Oct. 1926.

To Mr. H. Knowles,
10, Royal Arcade,
Cardiff.

W. Ellison.

Note. (1) This bill was sent by W. Ellison to H. Knowles, and H. Knowles agrees to pay the £250, by writing *Accepted, H. Knowles, 13th October* 1926 across the bill, and returns it to W. Ellison.

(2) The nominal day of payment is two months after 12th October, *i.e.* on 12th December, but three Days of Grace are now allowed, so that the bill is legally due on 15th December. The number of days for which the bill has to run = 19 (Oct.) + 30 (Nov.) + 15 (Dec.), *i.e.* 64 days.

(3) Days of Grace are not allowed on bills drawn "on demand" or "at sight."

(4) W. Ellison may now (i) hold the bill until 15th December and then obtain the £250 from Mr H. Knowles's bankers, or (ii) sell the bill to a banker or a bill broker.

(5) The bill will be worth £250 64 days hence, and so W. Ellison cannot expect to receive the full £250 on any day before 15th December.

(6) Theoretically, the bill broker ought to pay such an amount as would, when put out at interest for the unexpired period at an agreed percentage, produce £250.

(7) In practice, the bill broker pays the balance after deducting an agreed percentage for the unexpired period on the **whole** amount—£250. This deduction is known as **Banker's Discount** or **Commercial Discount**, and is reckoned in the same way as Simple Interest.

Example. A bill drawn by W. Ellison on 12th October for 2 months for £250 was discounted by a broker on 29th October at 8 per cent. Find the discount deducted by the banker and the sum paid for the bill.

Principal = £250 ; Rate = 8 per cent.

Date when bill is drawn is 12th October.

∴ Date when bill is due is 15th December.

∴ The unexpired period is from 29th October to 15th December, *i.e.* 2 + 30 + 15 days = 47 days.

The problem is to find the S.I. on £250 for 47 days at 8 per cent. per annum, and this sum is the Banker's Discount.

$$\text{S.I. or Banker's Discount} = \frac{£250 \times 8 \times 47}{100 \times 365}.$$
$$= £2,\ 11\text{s.}\ 6\text{d.}$$

$$\text{Sum paid to W. Ellison} = £250 - £2,\ 11\text{s.}\ 6\text{d.}$$
$$= £247,\ 8\text{s.}\ 6\text{d.}$$

Exercise 22

(Give answers to the nearest penny.)

1. Calculate the commercial discount on a B/E for £624, 13s. 8d. legally due 120 days hence at $2\frac{1}{4}$ per cent. per annum.

2. A B/E of face value £872, 18s. 8d. is discounted 65 days before it is legally due. If the rate charged is $6\frac{1}{2}$ per cent. per annum, find (*a*) the commercial discount, (*b*) the amount paid.

3. A B/E is *nominally* due on 27th March, and is discounted on 1st March at $4\frac{1}{2}$ per cent. per annum. If the face value of the bill is £2105, 17s. 6d., find the discount allowed.

4. A B/E drawn on 14th April was payable 5 months later. If the bill was for £872, 13s. 0d., find the money paid by a broker to the payee on 20th May. Rate $= 5\frac{3}{4}$ per cent. per annum.

5. How much cash can a tradesman obtain by discounting on 16th June a 3 months' bill for £700 drawn on 18th May, at $6\frac{1}{2}$ per cent. per annum ?

6. On 17th August a merchant drew a bill for £183, 14s. 4d. at 4 months. He discounted it on 30th September at $3\frac{1}{2}$ per cent. How much did his broker retain ?

7. How much must a banker pay on 14th October for a bill for £723 drawn 2nd August previously, at 4 months, if the current rate is 3 per cent. per annum ?

8. A B/E for £200, dated 12th July, at 3 months, was cashed by a banker at 5 per cent. per annum on 22nd August. What did the banker (i) deduct ? (ii) pay ?

9. What is the commercial discount on a bill for £850, 7s. 6d. payable at the end of 2 months at $5\frac{1}{2}$ per cent. per annum ?

10. What is the commercial discount on a bill for £185, 15s. 0d. discounted 75 days before legally due at $4\frac{7}{8}$ per cent. per annum ?

11. Find the banker's discount on a bill for £383 legally due on 31st May and discounted on 6th April at 5 per cent. per annum.

12. What is the commercial discount on a bill for £213, 6s. 8d. discounted 90 days before legally due at $4\frac{3}{4}$ per cent. per annum ?

13. Find the banker's discount on a bill for £450 legally due in 72 days, interest being at $7\frac{1}{2}$ per cent. per annum.

14. Find the banker's discount on a bill of £513, 14s. 0d., which is discounted at $6\frac{1}{2}$ per cent. per annum on 12th May, and is legally due on 12th June.

15. Find the banker's discount on a bill of £55, 10s. 0d. legally due in 53 days, if discounted at $4\frac{3}{4}$ per cent. per annum.

16. A bill for £1000 was drawn 19th March, payable six months hence, rate $3\frac{1}{2}$ per cent. per annum; it was discounted 12th June. What did the banker pay the holder of the bill?

Inverse Cases of Simple Interest

In the formula $\text{S.I.}=\dfrac{\text{P}\times\text{R}\times\text{T}}{100}$. There are four quantities which can vary, viz. Interest, Principal, Rate, and Time. When three of these four quantities are known, the fourth can be found, thus:

$$\text{P}=£\frac{100\times\text{S.I.}}{\text{R}\times\text{T}};\quad \text{R}=\left(\frac{100\times\text{S.I.}}{\text{P}\times\text{T}}\right)\text{ per cent.};\quad \text{T}=\left(\frac{100\times\text{S.I.}}{\text{P}\times\text{R}}\right)\text{ yrs.}$$

Example 1. *Find the sum which, invested for $3\frac{1}{2}$ years at 4 per cent. per annum, produces £18 as S.I.*

$$\text{P}=£\frac{100\times\text{S.I.}}{\text{R}\times\text{T}}=£\frac{100\times 18}{4\times 3\frac{1}{2}}=£\frac{900}{7}=£128,\ 11\text{s. }5\text{d.}$$

Example 2. *Find the rate per cent. per annum at which £360 produces £14, 10s. in 3 years.*

$$\text{R}=\frac{100\times\text{S.I.}}{\text{P}\times\text{T}}=\frac{100\times 14\cdot 5}{360\times 3}=1\cdot 3\text{ per cent. per annum.}$$

Example 3. *What length of time must £780 be invested at 4 per cent. per annum to produce £70, 4s. as simple interest?*

$$\text{T}=\frac{100\times\text{S.I.}}{\text{P}\times\text{R}}=\frac{100\times 70\cdot 2}{780\times 4\cdot 5}=\frac{7020}{3510}=2\text{ years.}$$

Exercise 23

1. At what rate per cent. per annum will the given principals amount to the given sums in the times stated:

(*a*) £129, 10s. 0d. to £145, 3s. 5d. in $2\frac{1}{2}$ years?
(*b*) £180 to £181, 13s. 9d. in 3 months?
(*c*) £400 to £409, 16s. 2d. in 179 days?
(*d*) £500 to £507, 2s. 6d. from 1st October to 8th February?

2. Find in how many years

(*a*) £2465 at $2\frac{3}{4}$ per cent. per annum will produce £112, 19s. 7d. as S.I.
(*b*) £3600 at $4\frac{1}{2}$ per cent. per annum will produce £135 as S.I.

(c) £2760 at $4\frac{1}{2}$ per cent. per annum will produce £31, 1s. 0d. as S.I.

(d) £245 at $2\frac{7}{8}$ per cent. per annum will produce £22, 17s. 10d. as S.I.

3. What principal invested for

(a) $2\frac{1}{2}$ years at 5 per cent. per annum produces £88, 10s. 0d. as S.I. ?

(b) $\frac{3}{5}$ year at $3\frac{1}{2}$ per cent. per annum produces £5, 17s. 1d. as S.I. ?

(c) 438 days at $4\frac{1}{2}$ per cent. per annum produces £108 as S.I. ?

(d) 146 days at 5 per cent. per annum produces £19, 12s. 4d. as S.I. ?

4. A man invests £350 in a bank and draws the interest annually. If the bank pays 3 per cent. per annum, when must the man close his account in order to be paid £404 in all ?

5. A man who borrows £50 agrees to pay $4\frac{3}{4}$ per cent. per annum, and he pays interest quarterly. In how many years will he have paid £2, 7s. 6d. ?

6. A merchant charges 5 per cent. per annum on an overdue account. If a customer owes him £1355 on 1st January 1925, when can he settle the account for £1524, 7s. 6d. ?

7. The annual cost of insuring property is 2s. per £100. In what time will as much money have been paid for insurance as the property is worth ?

8. £385 was lent on a mortgage at $5\frac{1}{2}$ per cent. per annum. In how many years will the total interest paid be £127, 1s. 0d. ?

9. To buy a house a man borrows £280 from a building society. In 7 months he repays the society £285, 16s. 8d. What rate of interest has he been charged ?

10. A widow wishes her annual income to be £278. What sum of money must she invest at $7\frac{1}{2}$ per cent. per annum to secure this income ?

11. In what time will £x double itself if lent at 5 per cent. per annum simple interest ?

12. Find the sum of money on which the S.I. for $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. is £189.

13. At what rate per cent. will £x double itself in 25 years ?

14. Mrs Jones pays 5s. each month as interest on £4. At what rate per cent. per annum is she paying interest ?

15. If I invest my money at $5\frac{1}{2}$ per cent. instead of at $6\frac{1}{2}$

per cent. per annum, I receive £72, 10s. 0d. less interest per year. How much money have I invested ?

16. A money lender lends money at $\frac{1}{2}$d. per week interest for every 2s. 6d. he lends. What rate per cent. per annum is this ?

17. How long must £225 be lent to produce £21 as simple interest at $3\frac{1}{2}$ per cent. per annum ?

18. On a bill for £530 legally due in 4 months the banker's discount is £13, 5s. 0d. What is the rate per cent. per annum ?

19. A borrower is paying £16, 10s. 0d. every half year for the loan of a certain sum of money. If the rate per annum is $5\frac{1}{2}$ per cent., what is the sum ?

20. If a borrower is paying 2s. 6d. per month as interest on £20, what rate per cent. per annum is he paying as interest ?

True Discount and Present Worth

In the example worked on p. 33 the discount is calculated to be £2, 11s. 6d., but *theoretically* it is not correct to deduct so much. Theoretically the amount deducted from a bill ought to leave such a balance as would amount to the full value of the bill if it were invested at the given rate of interest for the unexpired period.

In the example given, the balance

$$=£250-£2,\ 11s.\ 6d.=£247,\ 8s.\ 6d.,$$

and this balance at 8 per cent. per annum for 47 days would not amount to £250, but to £249, 19s. 6d.

If £247, 9s. 0d. were invested for 47 days at 8 per cent. per annum, it would amount to £250.

Hence the theoretical discount in the given example is £250 −£247, 9s. 0d.=£2, 11s. 0d. This discount is known as *True Discount.*

Suppose A owes B £104, due to be paid in 6 months, and A agrees to pay interest at the rate of 8 per cent. At this rate per cent. £100 becomes £104 in 6 months, hence A might settle the debt at once by paying £100 to B *now* instead of paying £104 at the end of the 6 months.

£100 is said to be the *True Present Worth* of the debt, and the difference between the amount of the debt and its True Present Worth is the *True Discount.*

∴ (1) The sum due = True Present Worth + True Discount.

(2) True Discount = Interest on True Present Worth.

Example (i). *The value of a building site will be* £2000 *in* 9 *months from to-day's date. What is its present worth if interest is reckoned at* 4 *per cent. per annum* ?

At 4 per cent. for $\frac{3}{4}$ year £100 would be worth £100 + £$(4 \times \frac{3}{4})$ = £103.

$$\text{If present worth of £103} = \text{£100,}$$

$$\text{Then} \quad ,, \quad ,, \quad ,, \quad \text{£2000} = \text{£}\frac{100 \times 2000}{103},$$

$$= \text{£1941, 14s. 11d.}$$

Example (ii). *Find the true discount on* £878, 15*s*. 0*d*. *due* 4 *months hence at* 6 *per cent. per annum.*

In $\frac{1}{3}$ year at 6 per cent. £100 will amount to £100 + £$(6 \times \frac{1}{3})$ = £102.

$$\text{If true discount on £102 is £2,}$$

$$\text{Then} \quad ,, \quad ,, \quad \text{£878·75 is £}\frac{878{\cdot}75 \times 2}{102}.$$

$$= \text{£17, 4s. 7d.}$$

EXERCISE 24

(Give answers to the nearest penny.)

1. A cargo of tobacco will be seasoned in 9 months, and will then be worth £1672. Find its present value if the rate of interest is reckoned at 6 per cent. per annum.

2. A man owns a field of wheat which he estimates will be worth £846, 6s. 0d. in 2 months' time. He sells it to-day and invests the money at $4\frac{1}{2}$ per cent. per annum, so that at the end of 2 months he will receive £846, 6s. 0d. Find his selling price.

3. Calculate the price (cash down) of a piano of the value of 55 guineas, allowing 4 months' credit at 5 per cent. per annum.

4. If 4s. 0d. is the difference between the simple interest and the true discount on a sum of money at 5 per cent. for 8 months, find the sum of money.

5. In 6 months' time a timber merchant will sell a stock of timber for £870, but to-day he will accept £750 for the stock. If £750 is its true present worth, find the rate per cent. per annum on which this amount is estimated.

6. A pawnbroker fixes £75, 15s. 0d. as the price for redeeming a suite of furniture 3 months hence. Find the true present worth of the goods if the rate is 4 per cent. per annum.

7. Find the present value of a consignment of cotton if it is estimated to be worth £10,000 in 8 months' time, interest being reckoned at 7 per cent. per annum.

8. Find the true discount that should be charged on a stock of oak beams worth £438 in 3 months if sold to-day, interest being reckoned at 6 per cent. per annum.

9. A farmer estimates that a field of barley will be worth £287, 10s. 0d. in 2 months' time. At what price must he sell it to-day, so that by investing the money he receives at 5 per cent. per annum he may obtain £287, 10s. 0d. at the end of the 2 months ?

10. In 8 months' time a consignment of ebony will be worth £307, 10s. 0d. Reckoning interest at $3\frac{3}{4}$ per cent. per annum, find its present worth.

11. Find the true discount on £515, 10s. 0d. payable in 7 months at 4 per cent. per annum.

12. Find the true present worth of £219, 5s. 0d. due in 3 months at 5 per cent. per annum.

13. Find the difference between the true and the banker's discount on £800 due 7 months hence at $3\frac{1}{2}$ per cent. per annum.

14. Find the difference between the true and the banker's discount on a bill of £330 drawn on 24th May for 2 months and discounted on 31st May at 7 per cent. per annum.

15. Find the true cash price of goods of the value of £3284, allowing 9 months' credit at $3\frac{1}{2}$ per cent. per annum.

16. What is the actual rate of interest which a banker receives for his money when he discounts a bill legally due in 6 months at 5 per cent. per annum, charging banker's discount ?

17. What is the present value of a stock of wine if it is calculated that four months hence it will be worth £8000, reckoning interest at $6\frac{1}{2}$ per cent. per annum ?

18. Find the true present value of a claim for £76, 18s. 6d. due at the end of 7 months. Rate $5\frac{1}{2}$ per cent. per annum.

19. Find the difference between the true and the commercial discount on £90, 15s. 0d. due in half a year at 10 per cent. per annum.

20. Find the true discount on £x payable in y years at z per cent. per annum.

Stocks and Shares

The word **Stock** has various meanings, for instance:

(1) **Stock** is the property which a merchant, a tradesman, or a company has invested in any business. It may include goods and money in hand, and goods and money due.

(2) Stock refers to money invested and is often called **Capital.** This stock or capital may be owned by one or by more persons jointly. If the stock be owned by more than one person, the amount of money comprising the stock is divided into shares, and the shareholders form a company.

(3) The money invested in a bank by shareholders is often called **Bank Stock**; similarly, money invested in railways is called **Railway Stock.**

(4) The money borrowed by Corporations on mortgage is designated **Corporation Stock.**

(5) Money lent to a Government constitutes a **National Debt.** This money is often spoken of as **Government Stock** or **The Public Funds,** and a fixed rate of interest is paid to the owners. Thus, we speak of "3 per cent. stock," meaning that the Government pays £3 per £100 on every £100 of this kind of stock.

In Great Britain the word stock is almost always used for amounts of £100 invested in Corporations or in the Public Funds.

The fundamental difference between investments (i) in the Post Office, in Personal or Public Loans, etc., and (ii) in Stocks and Shares, consists in the fact that the capital in the first case remains fixed, but the capital in the second case may vary. For instance, in to-day's newspaper we read that £100 Tasmanian $6\frac{1}{2}$ per cent. stock may be sold for £105; £100 Japanese 4 per cent. stock for £88; £100 Sheffield Corporation 6 per cent. stock for £105; £100 Edinburgh Corporation $4\frac{1}{2}$ per cent. stock for £96, and so on.

Sometimes Governments and Corporations arrange to repay the money borrowed on a certain date, and then we speak of such stock as **redeemable.** When no date for payment is fixed the stock is spoken of as **non-redeemable** stock.

When an owner of stock wishes to change it into cash he must sell it, and this is generally done through a stockbroker, who charges a fee known as commission or brokerage, which generally ranges from $\frac{1}{8}$ per cent. to $\frac{1}{4}$ per cent. on £100 **stock.**

A Limited Liability Company is a joint stock company in which the liability of its shareholders is restricted to the amount of unpaid share capital.

The original value of shares and stocks is spoken of as the **nominal** value or face value.

When stocks and shares are quoted at their nominal value they are said to stand **at par**; when quoted at more than their nominal value they are said to be **at a premium**; when quoted at less than their nominal value they are said to be **at a discount.**

Example 1. *If the £5 fully paid-up shares in a company are sold at* 10*s. premium, find* (i) *the number of shares I can buy for* £660; (ii) *the dividend on* 120 *shares at* 12 *per cent.*; (iii) *the actual rate of interest on my money.*

(i) Cost per share $= £5 + 10s. = £5{\cdot}5$.

$\therefore$ Required number of shares $= \dfrac{£660}{£5{\cdot}5} = 120$ shares.

(ii) Nominal value of 120 shares $= £5 \times 120 = £600$.

$\therefore$ Dividend $= 12$ per cent. of $£600 = £72$.

(iii) Rate per cent. $= \frac{7200}{660} = 10{\cdot}9$ per cent.

Example 2. *If I buy* £20 *Foundry Shares,* £18 *paid up, at* £2¼ *premium* (*brokerage* 9*d. per share*), *find* (i) *the cost of* 200 *shares;* (ii) *the half-yearly dividend on these shares at* 10 *per cent. per annum;* (iii) *the rate per cent. per annum I receive on my outlay.*

(i) Each share costs, Nominal Value + Premium + Brokerage

$= £18 + £2{\cdot}25 + £{\cdot}0375.$

$= £20{\cdot}2875.$

$\therefore$ 200 shares cost $£20{\cdot}2875 \times 200 = £4057{\cdot}5 = £4057$, 10s.

(ii) 200 shares, each £18 paid up, represents £3600 capital.

$\therefore$ Half-yearly dividend $= \frac{1}{2}$ of 10 per cent. of £3600.

$= £180.$

(iii) Interest per annum on $£4057{\cdot}5 = £180 \times 2$.

,, ,, ,, $£100 = £\dfrac{180 \times 2 \times 100}{4057{\cdot}5}$.

$\therefore$ Rate per cent. per annum on outlay $= 8{\cdot}9$ per cent.

Example 3. *How much War Loan* 5 *per cent. stock at* 107 *can be purchased for* £2600 ?

£107 in cash will purchase £100 stock.

$\therefore$ £2600 ,, ,, ,, $£\dfrac{100 \times 2600}{107}$ stock.

$= £2429{\cdot}907$ (correct to 3 places).

$= £2429$, 18s. 2d. (correct to nearest penny).

(The 5 per cent. describes the kind of stock and does not affect the calculation in this case.)

Exercise 25

1. What sum per annum is derived from the investment of £2968 in 4½ municipal stock quoted at 84 ?

2. A man bought 200 £1 shares when they were issued at par, and sold them later at a premium of 4s. 6d. each. Find his profit.

3. A corporation pays 3½ per cent. on its stock. Find the percentage rate of interest received by a man who bought this stock when it stood at 55.

4. £2793 is invested in 5½ per cent. Consols at 116⅜. Find

(i) the amount of stock purchased ; (ii) the annual income derived from the transaction.

5. £5 fully paid shares in a spinning company were quoted at 50s. below par. Calculate (i) the number of shares Mr Z. purchased for £250 ; (ii) the half-yearly dividend he received when the company declared a dividend of $3\frac{1}{4}$ per cent. per annum ; (iii) the rate per cent. interest he received on his investment.

6. A broker bought 850 £1 shares, 15s. paid up, for 18s.$10\frac{1}{2}$d. each. If the company later declared a dividend of 18 per cent. per annum, what was the actual rate per cent. per annum on the money invested ?

7. If a person by buying $3\frac{1}{2}$ per cent. stock realises $6\frac{1}{2}$ per cent. as interest on the money invested, how much below par is the stock quoted ?

8. A man sells out at 105 all his holding of £7200 in $4\frac{1}{2}$ per cent. war stock. The money received he reinvests in 4 per cent. stock quoted at 95. How will this affect his annual income ?

9. Mr X. buys 12,000 £10 shares fully paid up at £$11\frac{1}{8}$ each. What is the percentage interest on his outlay if the company declares a dividend of 12 per cent. per annum ?

10. The capital of a manufacturing company is £105,000, of which £60,000 ranks as $5\frac{1}{2}$ per cent. debentures and the rest as ordinary £5 shares. If the company makes a profit on the year's working of £8980, what is the highest possible dividend per share which can be declared on the ordinary shares ?

11. Find the half-yearly dividend Mr Allt receives from :

- (*a*) £500 in War Bonds 5 per cent.
- (*b*) £2500 in India Stock $2\frac{1}{2}$ per cent.
- (*c*) £750 in Egyptian Stock 3 per cent.
- (*d*) £450 in Sudan $5\frac{1}{2}$ per cent.
- (*e*) £250 in Glasgow Corporation $5\frac{1}{2}$ per cent.

12. Find the cash that must be paid to buy:

- (*a*) £500 stock in Australian $5\frac{1}{2}$ per cent. when the quotation is 102.
- (*b*) £350 stock in West Australia 6 per cent. at 103.
- (*c*) £950 stock in Uruguay 5 per cent. at 81.
- (*d*) 100 £ shares in Pengkalen Tin at $1\frac{3}{16}$.
- (*e*) 1000 £ shares, having 10s. paid up, when quoted at 5s. $7\frac{1}{2}$d.

13. Fifty persons join together to form a tennis club, with a capital of £90.

(*a*) If each person has an equal number of shares, how much does each one contribute ?

(*b*) If the shares are 1s. each, how many shares does each person hold ? How many shares are there altogether ?

(*c*) Mr D. sold his holding of 36 shares to Mr E. at a premium of 2d. each ; how much did he receive ?

(*d*) Mr F. sold his 36 shares to Mr H. when they were at a discount of 2d. ; how much did he receive ?

(*e*) At the end of the season there was found to be a profit of £13, 6s. 8d., and it was decided to put £6, 13s. 4d. to a reserve fund and to pay the remainder in dividend. How much did Mr E. receive in dividend if he held 36 shares at par and 36 for which he had paid 1s. 2d. each ? What was his percentage dividend ?

(*f*) What was the dividend (i) per share, (ii) per cent., on the original capital subscribed ?

(*g*) What rate per cent. did Mr H. receive, if he held 36 shares at par and 36 at 10d. each ?

14. (*a*) Find the brokerage at $\frac{1}{8}$ per cent. on the purchase of £500 stock of the British Guiana $5\frac{1}{2}$ per cent. stock at 103.

(*b*) Find the cost of this £500 stock, including brokerage.

(*c*) What is the actual percentage yield to the investor ? (Give answer to two decimal places.)

15. (*a*) How much London County Council $4\frac{1}{2}$ per cent. stock at $94\frac{1}{2}$ can be bought for £567 (excluding brokerage) ?

(*b*) Find the percentage return on the investment—to two decimal places.

16. (*a*) When £1 fully-paid shares are bought at 7s. 6d. premium, what is the selling price of 500 such shares ?

(*b*) What is the percentage return from this investment when a dividend of 10 per cent. is declared ?

17. (*a*) When Platt Brothers £1 shares are selling at 28s. 9d., find the cost of 200 of these shares.

(*b*) What rate per cent. is received on this investment when a dividend of 8 per cent. is declared ?

18. The difference between the incomes derived from investing a certain sum in $4\frac{1}{2}$ per cent. stock at $94\frac{1}{2}$ and 6 per cent. stock at 120 is £45. What amount was invested ?

19. A man spent £4750 in buying 4 per cent. stock at $94\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent. After receiving a half-yearly dividend he sold out at 96, brokerage again being $\frac{1}{8}$ per cent. Other expenses amounted to £4, 4s. 0d. What was his total profit ?

20. Mr Coe held £4000 stock in City of London 5 per cent. stock. He sold out at 103 and invested in New Zealand 6 per cent. stock at 108, sufficient to bring him in the same income as before. How much money had he remaining ?

TEST EXERCISE 26A

1. A thousand pounds is to be divided among A, B, and C. If A receives $\dfrac{2\frac{1}{4}}{6\frac{1}{4}}$ of it, B $\dfrac{3\frac{1}{3}}{8\frac{1}{3}}$ of the remainder, how much is left for C ?

2. How many tablets, each weighing $\frac{2}{5}$ of a Kg., can be made from 1 ton of soap ? How many ounces are left over ? (1 Kg.$=2{\cdot}2$ lbs.)

3. Draw a line $3{\cdot}6$ ins. long and divide it geometrically into 8 equal parts. Compare with the arithmetical result.

4. Find, to the nearest lb., the difference in the weight of $\cdot 45$ of a ton and the weight of $8{\cdot}3$ cwts.

5. $85\times 85=7225$. From this result write answers to the following: (*a*) $(8{\cdot}5)^2$; (*b*) $(8\frac{1}{2})^2$; (*c*) the area of a square having one side $8{\cdot}5$ ins.; (*d*) the price of $8\frac{1}{2}$ lbs. of meal at $8\frac{1}{2}$d. per lb.; (*e*) $8{\cdot}5\times\cdot 85$; (*f*) $7{\cdot}225\div\cdot 085$.

6. How much money is deposited yearly in the school bank if the average number of pupils is 256, the average weekly amount deposited is $7\frac{1}{2}$d., and the school is open 43 weeks per year ?

7. One-fifth of the amount that a barrel will hold is $6\frac{1}{2}$ galls. Find half the contents of the barrel and express the result as the decimal of 120 galls.

8. Find the cost of $\cdot 135$ of a ton of sugar at $2\frac{3}{4}$d. a lb., and give the answer to the nearest shilling.

9. (*a*) A train is travelling at a speed of $11\frac{1}{4}$ miles per hour. Express this speed in feet per second.

(*b*) A train is travelling at a speed of x miles per hour. Express this in feet per second.

10. A boy paid 52 weekly payments of 4s. for a bicycle. His friend bought one just like it for £7, 7s. 0d. in cash. How much per cent. more did the boy pay than his friend ?

TEST EXERCISE 26B

1. My watch gains 20 secs. per day of 24 hours. If it is put right at 9 a.m. on Monday, what time will it show at 4.30 p.m. on Friday ? Express the answer exactly to the fraction of a second.

2. Find the cost of 50 bales of cotton, each weighing 240 lbs., at $12\frac{15}{16}$d. per lb.

3. In one month 3,200,000 tons of coal were exported, and the value was £6,200,000. In another month 2,400,000 tons were exported, and the value was £4,000,000. Find the decrease in the price per ton.

4. A French merchant ordered, from England, goods to the value of £185 at a time when the exchange was 82·75 francs to the £, but did not pay for them until the exchange had risen to 88·5 francs to the £. How much would he have gained if he had paid for the goods at the time of ordering ?

5. Find the number of c.c. in 1 cu. in., assuming that 5 Kg. = 11 lbs., 1 cu. ft. of water weighs 62·3 lbs., and 1 litre of water weighs 1 Kg.

6. (*a*) Write down as decimals of £1 correct to three places : 12s. 8d. ; 15s. $4\frac{1}{4}$d. ; £8, 17s. 11d.

(*b*) Write in £, s. d. to the nearest penny, £0·713 ; £5·3786 ; £4·9531.

7. If the simple interest on a sum of money invested at $3\frac{1}{2}$ per cent. amounts to £12, 0s. 11d. in 7 months, find the sum invested.

8. A bill for £350 was drawn on 3rd March at 80 days. Reckoning interest at $3\frac{1}{2}$ per cent., calculate the value of the bill on 4th April.

9. A man deposits £20 every half-year at 3 per cent. per annum. Find the total value of his deposits at the end of 2 years from the date of the first deposit. Reckon compound interest added half-yearly.

10. A stockbroker invests for a client £292, 10s. 0d. in $2\frac{1}{2}$ per cent. Consols at $48\frac{7}{16}$, charging in addition $\frac{5}{16}$ per cent. on the amount of Consols bought. Find, to the nearest penny, the rate of interest the client will receive on the money he paid.

TEST EXERCISE 26C

1. Find, to two decimal places, the value of $\frac{74 \cdot 21 + 74 \cdot 3}{74 \cdot 3 - 74 \cdot 21}$.

2. Using the shortest methods you know, but showing all the steps, express as decimals of £1 :

(i) (*a*) 15s. $10\frac{1}{4}$d., (*b*) 13s. $9\frac{3}{4}$d., (*c*) £3, 9s. $7\frac{1}{2}$d., correct to three decimal places.

(ii) (*a*) 15s. $3\frac{3}{4}$d., (*b*) 18s. $10\frac{1}{2}$d., (*c*) £2, 10s. $11\frac{3}{4}$d. accurately.

3. Three successive readings on a gas-meter were 12,500 ; 45,400 ; 84,700. The charge for the amount consumed during the first interval was £4, 2s. 10d. Find, to the nearest penny, the cost for the second interval.

4. A dealer, on reducing the sale price of cloth by 10 per cent., found that at the lower price he sold 4 yds. more for £9. What was the reduced price per yard ?

5. Divide £24, 7s. $4\frac{1}{2}$d. in the ratio $\frac{1}{2} : \frac{1}{4} : \frac{1}{8}$.

6. Find the total interest at $2\frac{1}{2}$ per cent. per annum on £356 lent for 3 months, £777 lent for 4 months, and £396 lent for 5 months.

7. Find the cost in French currency of 415 galls. of turpentine at 8s. 6d. a gallon when £1 = 85·50 francs.

8. How much would a banker give on 21st May for a bill for £200 drawn 2nd April previously at 3 months, interest being at the rate of 4 per cent. per annum ?

9. Find the compound interest on £150 for 3 years at 3 per cent. per annum.

10. A contractor undertook to carry out an undertaking in 150 days. He employed 245 men, but after 80 days he found that only $\frac{7}{18}$ of the work had been done. How many extra men did he require for the work to be completed in the allotted time ?

TEST EXERCISE 26D

1. On 13th February 1922 the price of gold was £4·775 an ounce. What was the value in £, s. d. of 115 ounces ?

2. A manufacturer offered to sell an engine for £150, less 15 per cent., 5 per cent., and $2\frac{1}{2}$ per cent. successively. What would it cost the purchaser ?

3. A cycle dealer sold a bicycle for £12, 5s. 0d., making thereby a gross profit of $22\frac{1}{2}$ per cent. on the cost price. What was the cost price ?

4. Richard Barnsley borrowed £60 from a friend. At the end of 4 months he paid £61, 4s. 0d., which included principal and interest. At what rate per cent. per annum was the interest charged ?

5. A bill of exchange for £150 drawn on 1st March at 2 months was discounted on 12th March, the rate charged being 5 per cent. per annum. What was the amount of the discount ?

6. A rectangular plot 320 ft. long and 230 ft. wide has to be levelled and turfed. A contractor offered to do the work for 2s. 4d. a square yard. What was the amount of the estimate ?

7. Find the total amount of simple interest at $5\frac{1}{2}$ per cent. per annum on : £640 for 52 days and £585 for 81 days.

8. A stock of timber which will be ready for sale in 7 months will then be worth £2625. What is its present value ? (Reckon interest at the rate of 6 per cent. per annum.)

Test Exercise 26e

1. Work the following *mentally*, writing the answers *only* on your paper :

(*a*) $(2\frac{3}{10}\times 4)+(1\frac{7}{10}\times 4)$.
(*b*) Cost of 120 oranges at 3 for 2d.
(*c*) 5 per cent. of £8, 5s. 0d.
(*d*) 476×25 ; $47{\cdot}6\times 25$; $47{\cdot}6\times 2{\cdot}5$.
(*e*) Air space in cu. ft. of a room 16 ft. 6 ins. long, 10 ft. wide, and 9 ft. high.

2. It is estimated that a stock of goods will be worth £500 on 30th June. If interest is allowed for at the rate of $7\frac{1}{2}$ per cent. per annum, what is the cash value of the goods 4 months earlier ?

3. If 85·75 francs=£1, what is the value in francs of £150, 5s. 6d. ?

4. Write down the value of the following in £'s and decimals of a £ *exactly*—£4, 2s. $4\frac{1}{2}$d. and £8, 6s. 9d.

Also, in £, s. d., *correct within a farthing*—£6·42718 and £10·27638.

5. (*a*) How may bricks, each (including mortar) 9 ins. $\times$ $4\frac{1}{2}$ ins. $\times$ 3 ins., will be required to build a wall 57 ft. long, 9 ins. wide, and 7 ft. high ? and

(*b*) What will they cost at 75s. a thousand ?

(*Note.*—Reckon any number of bricks less than 100 as 100.)

6. A grocer buys butter at £11, 4s. 0d. a cwt. and sells it at a gross profit of $11\frac{1}{9}$ per cent. on the *selling* price. What is the selling price *per lb.* ?

7. What is the simple interest on £425 from 5th January 1926 to 30th May 1926 (both dates inclusive), at 6 per cent. per annum ?

8. What is the cost of 1 cwt. 3 qrs. of sugar if 2 cwts. 3 qrs. 7 lbs. cost £5, 12s. 6d. ?

9. A borrows £250 from B on 1st February 1926, and pays B at the end of 4 months £255, which amount includes principal and interest. At what rate per cent. per annum was the interest charged ?

10. A metal disc 6 ins. in diameter is reduced to a diameter of 5 ins. What is the area (top surface only) of the metal removed ? ($\pi=3\frac{1}{7}$.)

Test Exercise 26F

1. (*a*) Express the following *exactly* in £'s and decimals of a £ :

£4, 3s. $1\frac{1}{2}$d. ; £1, 4s. $7\frac{1}{4}$d. ; £2, 17s. $7\frac{1}{2}$d.

and (*b*) Calculate the cost of 745 yds. cloth at 3s. 9d. per yard.

2. Find, to the nearest yard, the length of a side of a square field whose area is $5\frac{1}{2}$ acres, and find the cost of fencing the sides of the field at 2s. 9d. per yard.

3. Three partners, A, B, and C, made a profit of £1753, 4s. 6d. A's share was one-third, B's share four-ninths, and C's share the remainder. How much did each receive ?

4. Find, correct within a penny, the cost of 2 cwts. 3 qrs. 17 lbs. butter at £10, 18s. 6d. per cwt.

5. Find, correct within a penny, the cost in £, s. d. of a yard of cloth worth fr. 4·75 per metre.

(1 yd.=·9144 metre ; fr. 25·22=£1.)

6. Find the *total* amount of simple interest at 6 per cent. per annum on :

£325 for 9 days ;
£730 ,, 51 days ; and
£500 ,, 32 days.

7. On 1st December 1925 Brenton & Maxwell gave G. Jordan their acceptance at 3 months for £420. Mr Jordan

discounts the bill on 22nd January 1926, his banker charging him at the rate of 6 per cent. per annum.

(a) Explain as clearly as you can what you understand these transactions to mean; and

(b) Calculate the amount Mr Jordan receives.

8. A grocer buys 6 cases of oranges at 21s. 3d. per case, each case containing 240. If 6 per cent. of the oranges are unsaleable, at what price per dozen must he sell the remainder so that he may make a gross profit of about 25 per cent. on cost price?

9. A picture dealer bought a picture for five hundred guineas in cash. He kept the picture for 2 months and then sold it for £600, the purchaser also paying cash. Assuming that it would be fair to allow 15 per cent. per annum on the purchase money for interest, insurance against loss, etc., what was the dealer's net profit?

10. The net amount of an invoice, after a discount of 20 per cent. has been deducted, is £4, 10s. 8d. What was the gross amount?

SECTION II

ALGEBRA

Revision

EXERCISE 27

1. The speed of an aeroplane was twice that of a motor car. If the motor travelled at x miles per hour, express the speed of the aeroplane in feet per second.

2. The volume of concrete in a certain pipe is ·79 of the length times the difference between the squares of the external and internal diameters. Write a general statement of this fact. (Use the letters V, l, D, and d.)

3. The weight in hundredweights which a beam will bear is given by the formula $\frac{21d^3}{20L}$, where d = depth of the beam in inches, and L = the length of the beam in feet. Find the weight a beam will bear if it is 70 ft. long and 10 ins. in depth.

4. A boy owns £x, $2y$s. 0d. and owes £$\frac{x}{2}$, ys. 6d. Into how many sixpences can he change the balance after paying his debt ?

5. Simplify (i) $3a(b+c)-2(b-c)+b(a-c)+5ab$.
(ii) $2x(y-x)-7y(x-y)+48$.

6. The 4 trucks making up a train weigh a tons $2b$ cwts. c lbs. ; $2b$ tons a cwts. ; $3c$ tons a cwts. $2b$ lbs. ; $5c$ tons. Find in lbs. the total weight of the trucks.

7. Add $7x^3+5x^2y$; $11x^2y-9xy^2+y^3$; $4x^3-7y^3$; $3x^2y-11xy^2+6y^3$.

8. Subtract $\frac{x^2}{2}+3x+4$ from $2x^2-\frac{5x}{2}-7$.

9. From A to B is $\frac{1}{2}x$ miles, from B to C is $14y$ miles. How far has a man journeyed after going from A to C and back $12p$ times ?

10. A brick is $9x$ ins. long, $4\frac{1}{2}x$ ins. wide, and $3x$ ins. thick. What number of bricks will build a rectangular stack $16x$ yds. $\times 9x$ yds. $\times 4\frac{1}{2}x$ yds.?

11. Mother is 30 years older than her daughter. Twenty years hence she will be twice as old as her daughter. Find the daughter's present age.

12. Harry has 6 more marbles than John. If Harry gave one-third of his marbles to John, John would then have 10 more marbles than Harry. How many has each?

13. Solve for y: $\frac{y}{x} = mn$; $\frac{y}{4} - 3 = 1$; $\frac{p}{q}y = x$; $\frac{y}{3} + 6 = 9$; $\frac{2y-2}{6} = \frac{y}{4}$.

14. Find three consecutive numbers, such that if the first is divided by 2, the second by 3, and the third by 5, the sum of the three quotients is 133.

15. Twice the excess of 56 over three times a given number equals four times the given number plus 2. Find the number.

16. A bill for £7, 10s. 0d. was paid in half-crowns and florins. If 70 coins were used, how many were there of each kind?

17. Take a certain number, x. Then (i) add 4 to x and divide the result by 3; (ii) deduct 2 from x and divide the result by 2; (iii) add 6 to x and divide the result by 2. If the sum of the three quotients is 14, find the value of x.

18. Anthracite yields the following products when burned: carbon, 92·5 per cent.; hydrogen, 3·5 per cent.; oxygen, 2·5 per cent.; ash, 1·5 per cent. Draw on squared paper a chart to illustrate the composition of anthracite.

19. If 1 kilogram $= \frac{11}{5}$ lb., draw a graph showing the relationship between lbs. and kilograms, and from the graph find the value of 1 lb. in kilograms and of 8 kilograms in lbs.

20. The average daily temperatures in a certain town during the separate months of the year were as follows:

Month	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Temp.	39	39	42	47	53	60	63	62	57	50	44	40

Plot a graph to show the variations of the average temperature throughout the year.

Multiplication

Example (i). *Multiply* $7x+9$ *by* $3x+5$.
,, (ii). ,, $3x+8$ *by* $5x-3$.

$$\begin{array}{rr}
\multicolumn{2}{c}{\text{(i)}} \\
7x & +9 \\
3x & +5 \\
\hline
21x^2+27x & \\
+35x & +45 \\
\hline
21x^2+62x & +45 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rr}
\multicolumn{2}{c}{\text{(ii)}} \\
3x & +8 \\
5x & -3 \\
\hline
15x^2+40x & \\
-9x & -24 \\
\hline
15x^2+31x & -24 \\
\hline
\end{array}$$

Study the two following examples of multiplication, of which the first is a shortened form of the second :

$$\begin{array}{r}
\text{(i)} \\
212 \\
13 \\
\hline
212 \\
636 \\
\hline
2756 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
\text{(ii)} \\
2(10)^2+1(10)+2 \\
1(10)+3 \\
\hline
2(10)^3+1(10)^2+2(10) \\
+6(10)^2+3(10)+6 \\
\hline
2(10)^3+7(10)^2+5(10)+6 \\
\hline
\end{array}$$

In the first case the powers of 10 are omitted and the coefficients only are used. We may often shorten the working in algebraic multiplication by using the coefficients only.

EXERCISE 28

Expand the following :

1. $(a+2b)\ (a+3b)$.
2. $(x+y)\ (x+y)$.
3. $(x+y)\ (x-y)$.
4. $(x-y)\ (x-y)$.
5. $(2a-3b)\ (4a+3b)$.
6. $(5x-2)\ (6y-4)$.
7. $(3x-8)\ (4x-2)$.
8. $(4x-3)\ (2x+5)$.
9. $(3x+2y)\ (4x-3y)$.
10. $(a+b+c)\ (a+b)$.

EXERCISE 29

Multiply:

1. $a^2-2ab+2$ by $a+2b$.
2. $2a^2-3b+2$,, $2a+7$.
3. $2a^2+a+6$,, $a-5$.
4. x^2+2x-6 ,, x^2+2.
5. $a^2+2ab+4$,, $a+b$.
6. $x^2+11x+12$ by $1-x$.
7. a^2b+2ab ,, $a-3$.
8. a^3-2a^2+a-1 ,, $2-a$.
9. $3x^2+4x+2$,, $3x^2+x$.
10. $2x^2+7x-6$,, x^2-1.

Multiply:

11. x^2+2x+2	by $x-1$.	**16.** $7a^2-7a+2$ by $6a^2+a+1$.
12. a^3+2a+3	,, a^3+a^2.	**17.** a^3-2a^2+a+1 ,, $3a+2$.
13. $6p^3+8p^2-p$	,, $2p-1$.	**18.** $17x^3+4x^2+2$,, $4x-3$.
14. $4x^2+3x-3$	,, $2x^2-x$.	**19.** x^2+3x+2 ,, $x-1$.
15. $3a-2$	,, a^2+a.	**20.** $3y^4+2y^3+y$,, $1-2y$.

21. A rectangle is $(3x+2y)$ ins. by $(4x-y)$ ins. Find (i) its area ; (ii) by how much the area is increased if each side is increased by y ins.

22. What is the difference in volume between a cube having an edge $(m+2)$ ins. in length and one whose edge is 5 ins. less ?

23. When x boys and $2y$ girls each have $(3x-y)$ apples, how many apples have they altogether ?

24. A stick is x yds. y ft. in length. Find, in feet, the total length of $(4x+3y+2)$ similar sticks placed end to end.

25. The area of a triangle is $\frac{B\times H}{2}$. Find the area of a triangle having $B=2(4p+q)$ ins. and $H=(2p-q)$ ins. By how much does the area differ from that of the square on B ?

26. A newsboy delivers daily $48a$ magazines and $102b$ newspapers one at a time. Find his total number of deliveries in a weeks and 2 days.

27. A tap A fills a cistern at the rate of x galls. per hour. A tap B empties the cistern at a rate of $2y$ galls. per hour. If both taps run together, the tank is full in y hours. How many gallons remain to be added after $(y-6)$ hours ?

28. If a doctor examines daily x men, y women, and $3x$ children, how many patients does he examine in $(3y-2)$ days ?

29. The quotient of a division sum is $x-3b$, the divisor is $x+2b$, and the remainder is $2b$. Find the dividend.

30. A square paddock has a side $11x$ yds. m ft. in length. Find, in feet, the length of fencing required for $(m-3)$ separate paddocks of the same dimensions.

Division

Example. *Divide* $7x^2+7x^3+12-8x$ *by* $x+2$.

$$
\begin{array}{rl}
x+2)\!\!\!\! & 7x^3+7x^2-8x+12\,(7x^2-7x+6 \\
 & \underline{7x^3+14x^2} \\
 & \quad -7x^2-8x+12 \\
 & \quad \underline{-7x^2-14x} \\
 & \qquad\qquad 6x+12 \\
 & \qquad\qquad \underline{6x+12}
\end{array}
$$

Study the following examples of division, of which the first is a shortened form of the second:

(i)	(ii)
$123)2583(21$	$1(10)^2+2(10)+3)2(10)^3+5(10)^2+8(10)+3(2(10)+1$
246	$2(10)^3+4(10)^2+6(10)$
123	$+1(10)^2+2(10)+3$
123	$+1(10)^2+2(10)+3$
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$

In the first case the powers of 10 are omitted and the coefficients only are used. As in multiplication, we may often shorten the working in algebraic division by using the coefficients only.

Exercise 30

Divide:

1. x^2+y^2+2xy by $x+y$.
2. x^2+y^2-2xy by $x-y$.
3. a^2-b^2 by $a+b$.
4. $7x^2-4xy-3y^2$ by $7x+3y$.
5. $2a^2-a^2b+2ab-ab^2$ by $ab-2a$.
6. $a^2-2a^3b+a^4b^2$ by a^2-a^3b.

Divide:

7. $14x^2-8x-6$ by $2x-2$.
8. $9p^2q+12pq^2-6pq-21p-28q+14$ by $3pq-7$.
9. $12x^2-17xy+6y^2$ by $3x-2y$.
10. x^3+3x^2-3x-1 by $x-1$.
11. a^3+2a^2-3a by a^2-a.
12. $4x^3-3x^2-3x+2$ by $x-1$.

Divide:

13. $42a^2+28ab+18a+12b$ by $6a+4b$.
14. $24x^2-13xy-2y^2$ by $3x-2y$.
15. $6a^2-ab-b^2$ by $2a-b$.
16. $56p^2+32pq^2-7p-4q^2$ by $7p+4q^2$.
17. $36xy-24x-9y^2+6y$ by $3y-2$.
18. $26m^2+49mn-6n^2$ by $m+2n$.

19. Find the quotient and remainder when $a^4+6a^3-20a^2+b$ is divided by a^2-3a+2. By how much must the dividend be increased so as to give no remainder?

20. A rectangle has an area of $(2m^2-5mn+2n^2)$ sq. ins. If the base is $(2m-n)$ ins., find the altitude. Check the result when $m=5$ and $n=1\frac{1}{2}$.

21. A rectangular metal plate has an area of $(49l^2-16b^2)$ sq. ft.; the length is $(7l+4b)$ ft. Find the breadth.

22. What is the length of carpet $(3x+5)$ ft. in width required to cover a rectangular floor having an area of $(12x^2+38x+30)$ sq. ft.?

23. The circumference of a wheel is $(7x+y)$ ins. How many complete revolutions will it make in travelling $(21x^2-11xy-2y^2)$ yds.?

24. A glazier has a sheet of glass $(30x^2+7x-2)$ sq. ft. in area. What is the greatest number of windows $(6x-1)$ sq. ft. in area into which he can cut it?

Simple Equations

A statement that two quantities are equal is an equation. If the equation does not contain any higher power of the unknown quantity than the first it is called a *Simple Equation*.

I. The Unknown quantity on both sides.

Example. $5x=18-2x+3.$

Then $5x-2x=18+3.$

And $3x=21.$

$\therefore \quad x=7.$

Steps

(i) Collect all the terms containing the unknown quantity on the left-hand side (L.H.S.) and all the known terms on the right-hand side (R.H.S.).

(ii) Be sure to change the sign of a term when transferred to the other side of the equation, and be ready to explain why this is done.

(iii) Find the algebraic sum of each side.

(iv) Find the value of the unknown quantity.

Exercise 31

Solve the following:

1. $4x+2=3x+14.$
2. $27+7x=5x+29.$
3. $6y+4y+8y=3y+105.$
4. $30-x=52-2x.$
5. $20p-31p+106=118-23p.$
6. $15ab-57=25-26ab.$
7. $6r-18-4r+25=r-14.$
8. $17t+24-14t+2=52-10t.$
9. $7x+9x+12x=13x+10x-5x+120.$
10. $16y+13y+2y=y+240.$
11. A shopkeeper has £2, 12s. 0d. in his cash box. The money is in pence, shillings, and half-crowns. The number of shillings is three times the number of half-crowns, but only one-fourth the number of pence. Find how many coins there are of each kind.

12. Four times a certain number, together with 5, equals half the number together with 12. Find the number.

13. The difference between two numbers is 8. Twice the greater number exceeds four times the smaller by 10. Find the numbers.

14. A lady pays £2, 15s. 0d. for 30 yds. of muslin and 40 yds. of cloth. If the cloth is twice as valuable as the muslin, find the cost of a yard of each.

15. A tank containing water will be filled if 42 galls. are added. The tank will then contain 7 times as much water as at first. What is the total capacity of the tank ?

II. Containing Bracketed Terms.

Remove the brackets first. After collecting the terms proceed as already shown.

Example. $6(x-3)-4(3x+2)=6x-50.$

Then $6x-18-12x-8=6x-50.$

And $-6x-26=6x-50.$

And $-12x=-24.$

$\therefore$ $x=2.$

Check: L.H.S. $=$ R.H.S.

$\therefore$ $6(2-3)-4(3\times2+2)=6\times2-50.$

Then $-6-32=12-50.$

And $-38=-38.$

EXERCISE 32

Find the value of the unknown quantity :

1. $3(2x+6)-(2x+4)=3x-5.$

2. $6(x+3)-(x-2)=3x-2.$

3. $x(x-3)=x^2-9.$

4. $6(x+10)-(6+4x)=x(3-8)+19.$

5. $4(3x-x)+3(-x+1-6)=0.$

6. $10(10+2x+1)=19x+96.$

7. $7(x-2)+2x=5(x-1)+x.$

8. $5(x+1)+6(x-2)=7(x-1)+24.$

9. $27(y+5)-10(y+10)=6(y-8)-16.$

10. $31(s+2)+2(s-13)=29(s+1)+7.$

11. Divide 18 into two parts, so that three times the smaller together with five times the larger part may equal 78.

12. A newsagent pays wages to 150 boys. He pays 2s. each to some and 5s. each to the rest. To how many does he pay 2s. if his total wage bill is £24 ?

13. A lady bought 39 books. For some she paid 2s. each, and for the rest 2s. 6d. each. If she spent £4, 6s. 6d. altogether, how many of each did she buy?

14. In a shooting competition a boy gains 5 marks for each "bull" he scores and loses 3 marks each time he fails to do so. After 16 shots he has no marks. How many times did he score?

15. A farmer bought 12 cows and 6 horses for £768. If a horse cost £8 more than a cow, find the cost of one of each.

III. Involving Fractions.

Convert to the usual form by multiplying both sides by the L.C.M. of the denominators. Then proceed as already shown.

Example. $\dfrac{4}{5x+2}=\dfrac{5}{4x-2}$.

Multiply by $(5x+2)(4x-2)$, *i.e.* by the L.C.M. of the denominators.

Then $4(4x-2)=5(5x+2)$.

And $16x-8=25x+10$.

And $-9x=18$.

$\therefore$ $x=-2$.

Exercise 33

Solve the equations:

1. $\dfrac{1}{3}\left(\dfrac{7a}{2}-6\right)+\dfrac{7a+1}{6}=3a-\frac{1}{2}$.

2. $\dfrac{x-6}{7}+\dfrac{2x-3}{3}=3(x-5)$.

3. $\dfrac{a+3}{5}+\dfrac{a+5}{3}=3$.

4. $\dfrac{3y}{2}+\dfrac{7y}{3}+\dfrac{7y}{3}=37$

5. $\dfrac{2+a}{3}-\dfrac{6a-2}{4}=\dfrac{1}{2}$.

6. $\dfrac{1}{p}+\dfrac{3}{p-1}=\dfrac{4}{p(p-1)}$.

7. $\dfrac{1}{5}-\dfrac{x}{10}-\dfrac{1}{2}\left(3-\dfrac{x}{2}\right)=\dfrac{1}{4}\left(10+\dfrac{x}{5}\right)$.

8. $4(x-2)-5(11-x)=3(2x-11)-2x$.

9. $\frac{1}{4}(5x+1)-\frac{1}{3}(6x-1)-\frac{1}{6}(3x-1)=0$.

10. $\dfrac{9}{x-2}-\dfrac{7}{x}=\dfrac{8}{x}$.

11. $(a-3)(x-a)=a(a-x-3)-39$.

12. $\dfrac{3x}{2}-\dfrac{x+4}{5}=\dfrac{2(2x-7)}{5}$.

13. In making a journey of 56 miles a cyclist travels one

portion at the rate of 10 miles per hour and another portion at the rate of 14 miles per hour. How long does he travel at 10 miles per hour if the whole journey takes 5 hrs. ?

14. A tourist walked to a village at the rate of 2 miles per hour. He returned by another route which was 4 miles longer and reduced his speed by $\frac{1}{2}$ mile per hour. His complete journey took 12 hrs. How far did he walk ?

Expanding and Factorising

Type I. $a(x+y+z)=ax+ay+az$ . . . . . (i)

$ax+ay+az=a(x+y+z)$ . . . . . (ii)

(i) is an example of expanding and (ii) is an example of factorising.

Fig. 3 illustrates this type graphically.

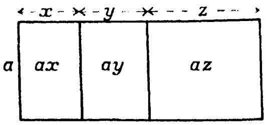


FIG. 3.

When an expression is exactly divisible by a quantity, that quantity is a factor of the expression.

Other Examples. $a^2b+ab+2ab^2=ab(a+1+2b)$.
$4x+2x^2+8x^3=2x(2+x+4x^2)$.
$m^2(x+y)+n(x+y)=(m^2+n)(x+y)$.

EXERCISE 34

Resolve the following expressions into factors :

1. $ab+ac$.
2. $6x^2-x$.
3. $2x^3+6x^2+2x$.
4. $24-32y^2$.
5. $-18x^3-9x^2-9x$.
6. $apxy+3x^2yp+6apy$.
7. $25x^3+5x^2y+50x^2y^2$.
8. $56x^2y^2z+32x^2yz-40x^2yz^2$.
9. $13p^2q-26pq^2+39q^3$.
10. $7a^2x^3-21a^2bx^2+56a^2bcx^2$.
11. $x(a+b)+x(d+2)$.
12. $a(x+y)+a^2$.
13. $(a+b)^2+(a-b)(a+b)$.
14. $(x+3)^2-3(x+3)$.
15. $m^2(n-c)+3m^2$.
16. $6xy(a+2)-3(a+2)$.
17. $pqr^2-p^2qr+pq^2r$.
18. $ab(c-z)-ab(c-3)$.
19. $ca(b+3)-ac(b-3)$.
20. $(m+n)^3+6(m+n)^2$.

Type II. $\quad (a+b)^2=a^2+2ab+b^2.$
$\quad a^2+2ab+b^2=(a+b)^2.$

We may illustrate this graphically (fig. 4).

The whole figure represents the square of $(a+b)$. It is composed of a square $=a^2$, a square $=b^2$, and two rectangles each equal to ab.

Hence $(a+b)^2=a^2+b^2+2ab$.

This result may be stated thus:

The square of the sum of two terms equals the square of the first **plus** *twice their product* **plus** *the square of the second.*

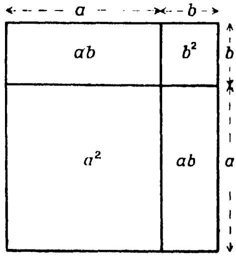


FIG. 4.

Example (i). *Expand* $(3x+4y)^2$.

$(3x+4y)^2=(3x)^2+2(3x)(4y)+(4y)^2.$
$\quad =9x^2+24xy+16y^2.$

Example (ii). *Find the value of* $100{\cdot}5^2$.

$100{\cdot}5^2=(100+{\cdot}5)^2.$
$\quad =100^2+2\times{\cdot}5\times100+{\cdot}5^2.$
$\quad =10{,}000+100+{\cdot}25.$
$\quad =10{,}100{\cdot}25.$

EXERCISE 35

Expand the following:

1. $(m+p)^2$.	**6.** $(10x+10y)^2$.	**11.** $\left(\frac{x}{2}+\frac{y}{3}\right)^2$
2. $(p+s)^2$.	**7.** $(mn+pq)^2$.	**12.** $(1+\frac{2}{3}x)^2$.
3. $(a+4)^2$.	**8.** $(8a+3b)^2$.	**13.** $\left(a+\frac{x}{4}\right)^2$.
4. $(2a+3)^2$.	**9.** $(2ab+3bc)^2$.	**14.** $\left(\frac{m}{8}+\frac{n}{4}\right)^2$.
5. $(3b+4c)^2$.	**10.** $(4cd+10)^2$.	**15.** $\left(ab+\frac{b}{3}\right)^2$.

Evaluate:

16. $(10{\cdot}5)^2$; $(8\frac{1}{2})^2$; $(100{\cdot}4)^2$.

17. $(11\frac{1}{2})^2$; $(7\frac{3}{4})^2$; 1009^2.

18. Find the square of 1987 if the square of 1986 is 3,944,196.

19. Find the area of a square having one side $(2a+\frac{1}{5}b)$ ins. in length.

20. Find the volume of a square prism on a base $(x+2y)$ ins. long and having a height of 3 ins.

The expression $a^2+2ab+b^2$ is said to be a perfect square.

When any expression has the form of a perfect square it may be factorised as the square of a binomial.

Example. Factorise $25p^2+40pq+16q^2$.

$$\begin{aligned}\text{Expression} &= 25p^2+40pq+16q^2.\\ &= (5p)^2+2(5p)(4q)+(4q)^2.\\ &= (5p+4q)^2.\end{aligned}$$

Exercise 36

Factorise the following:

1. $p^2+2pq+q^2$.
2. $4y^2+12y+9$.
3. $4a^2+8ab+4b^2$.
4. $x^2y^2+2mnxy+m^2n^2$.
5. $16d^2+72cd+81c^2$.
6. $r^2t^2+2rtq+q^2$.
7. $144a^2+240ab+100b^2$.
8. $36x^2+60xy+25y^2$.
9. $49+42q+9q^2$.
10. $100+40a+4a^2$.

Find the factors of the following perfect squares:

11. $a^2+\dfrac{4a}{b}+\dfrac{4}{b^2}$.
12. $16+\dfrac{8}{x}+\dfrac{1}{x^2}$.
13. $\dfrac{a^2}{4}+a+1$.
14. $\frac{1}{4}y^2+\frac{1}{2}xy+\frac{1}{4}x^2$.
15. $\dfrac{x^2}{16}+x+4$.
16. $\dfrac{a^2}{9}+\dfrac{4a}{3}+4$.
17. $\frac{1}{4}a^2+\frac{1}{4}ab+\frac{1}{16}b^2$.
18. $100x^2+2xy+\frac{1}{100}y^2$.
19. $1+\frac{1}{3}a+\frac{1}{9}a^2$.
20. $\dfrac{m^2}{16}+2mn+16n^2$.

Type III. $\quad (\mathbf{a}-\mathbf{b})^2=\mathbf{a}^2-\mathbf{2ab}+\mathbf{b}^2.$

$$(\mathbf{a}^2-\mathbf{2ab}+\mathbf{b}^2)=(\mathbf{a}-\mathbf{b})^2.$$

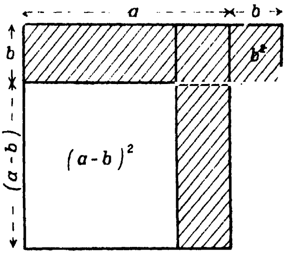


Fig. 5.

We may illustrate this graphically (fig. 5).

The whole figure $=a^2+b^2$. It is composed of an unshaded portion, $(a-b)^2$, and a shaded portion, $2ab$.

Hence $(a-b)^2=a^2+b^2-2ab.$
$\qquad\qquad\qquad\;\; =a^2-2ab+b^2.$

This result may be stated thus:

The square of the difference of two terms equals the square of the first **minus** *twice their product* **plus** *the square of the second.*

Example (i). *Expand* $(3x-4y)^2$.

$$\begin{aligned}(3x-4y)^2 &= (3x)^2-2(3x)(4y)+(4y)^2.\\ &= 9x^2-24xy+16y^2.\end{aligned}$$

Example (ii). *Find the value of* 98^2.

$$98^2=(100-2)^2.$$
$$=100^2-2\times2\times100+2^2.$$
$$=10,000-400+4.$$
$$=\underline{\underline{9604}}.$$

Exercise 37

Expand the following. Write the answers only :

1. $(x-y)^2$; $(a-b)^2$; $(c-d)^2$.
2. $(2a-3b)^2$; $(a-6)^2$; $(7-5a)^2$.
3. $(m-n)^2$; $(7a-5)^2$; $(3p-q)^2$.
4. $(a^2-b^2)^2$; $(m^2-3)^2$; $(p^2-q)^2$.
5. $(1-x)^2$; $(3-m)^2$; $(4-pq)^2$.
6. $(a^2-4b)^2$; $\left(\frac{a}{2}-\frac{b}{2}\right)^2$; $\left(3p-\frac{q}{2}\right)^2$.
7. $(xy-1)^2$; $(2y-z)^2$; $(r-8)^2$.
8. $(m-3n)^2$; $(3r-s)^2$; $(pq-r)^2$.
9. $(x-5)^2$; $(m-3)^2$; $(n-1)^2$.
10. $(4y-2)^2$; $(3m-2n)^2$; $(5y-3z)^2$.

Find the value of :

11. 97^2 ; 76^2 ; 199^2 ; 19^2 ; 59^2 ; 148^2.
12. 39^2 ; 68^2 ; 88^2 ; 119^2 ; 194^2 ; 498^2.

Since $a^2-2ab+b^2$ is a perfect square, any expression of this form may be factorised.

Example. Factorise $4x^2-4ax+a^2$.

Expression $=(2x)^2-2(2x)(a)+(a)^2$.
$=(2x-a)^2$.

Exercise 38

Factorise the following :

1. $1-14p+49p^2$.
2. $x^6-2x^3y^3+y^6$.
3. $\frac{1}{16}x^2-\frac{3}{2}xy+9y^2$.
4. $4p^6-6p^3q^2+\frac{9}{4}q^4$.
5. $1+4x^2-4x^4$.
6. $9x^2-12xy+4y^2$.
7. a^2-2a+1.
8. m^2-6m+9.
9. $a^2b^2-2abcd+c^2d^2$.
10. $25a^2-80ab+64b^2$.
11. $1-10k^2+25k^4$.
12. $100x^2-20x+1$.
13. $\frac{1}{9}x^2-\frac{1}{6}xy+\frac{1}{16}y^2$.
14. $9-\frac{6}{a}+\frac{1}{a^2}$.
15. $\frac{a^2}{4}-a+1$.
16. $\frac{m^2}{9}-\frac{4m}{3}+4$.

17. $16b^2-24ab+9a^2$.
18. $\frac{a^2}{9}-\frac{ab}{3}+\frac{b^2}{4}$.
19. $p^6-4p^3q^2+4q^4$.
20. $100-100a+25a^2$.

Type IV. $(\mathbf{x}+5)(\mathbf{x}+3)=\mathbf{x}^2+3\mathbf{x}+5\mathbf{x}+15=\mathbf{x}^2+8\mathbf{x}+15.$
$(\mathbf{x}-5)(\mathbf{x}-3)=\mathbf{x}^2-3\mathbf{x}-5\mathbf{x}+15=\mathbf{x}^2-8\mathbf{x}+15.$
$(\mathbf{x}+5)(\mathbf{x}-3)=\mathbf{x}^2-3\mathbf{x}+5\mathbf{x}-15=\mathbf{x}^2+2\mathbf{x}-15.$
$(\mathbf{x}-5)(\mathbf{x}+3)=\mathbf{x}^2+3\mathbf{x}-5\mathbf{x}-15=\mathbf{x}^2-2\mathbf{x}-15.$

Note the products:

(i) The first term is always x^2.
(ii) The second term is always x times the algebraic sum of the second terms of the multiplier and multiplicand.
(iii) The third term is always the product of the second terms of the multiplier and multiplicand.

Exercise 39

Expand the following:

1. $(x+4)(x+2)$.
2. $(x-6)(x-3)$.
3. $(x+5)(x-1)$.
4. $(x-6)(x+3)$.
5. $(x+8)(x-5)$.
6. $(x-1)(x-3)$.
7. $(x-1)(x-2)$.
8. $(x-3)(x-4)$.
9. $(x+5)(x+5)$.
10. $(x+2)(x+2)$.
11. $(x-6)(x-6)$.
12. $(x-2)(x+1)$.
13. $(x-5)(x-2)$.
14. $(x+6)(x-2)$.
15. $(x+7)(x+1)$.
16. $(x+7)(x-1)$.
17. $(x-7)(x+1)$.
18. $(x-9)(x+9)$.
19. $(x+3)(x+8)$.
20. $(x-3)(x-8)$.
21. $(x-3)(x+8)$.

To factorise we proceed thus:

Example. Factorise $x^2+8x+15$.

[Mental work:
(i) The first term of each factor must be x.
(ii) The second terms of the factors are two numbers having their sum $+8$, and their product $+15$.
(iii) $+3$ and $+5$ are the required numbers.]

$\therefore$ Result $=(x+3)(x+5)$.

Exercise 40

Factorise the following:

1. x^2+6x+8.
2. $a^2+8a+15$.
3. $b^2+2b-35$.
4. $a^2-4a-60$.
5. $x^2-4x-45$.
6. $x^2-12x+35$.
7. $m^2+13m+42$.
8. $a^2-7ab-8b^2$.
9. $x^2-5xy-24y^2$.
10. $p^2+3pq+2q^2$.

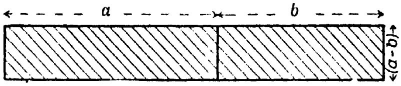
Resolve into factors:

11. $a^2+10a+24$.
12. $x^2+15x+56$.
13. $a^2-7a+10$.
14. $p^2-8pq+15q^2$.
15. $a^2b^2-12ab+32$.
16. $x^2y^2-16xyz+15z^2$.
17. $c^2+17c-38$.
18. $x^2+3x-28$.
19. $m^2n^2-4mn-32$.
20. $a^4b^4+a^2b^2-12$.

Type V. $\mathbf{(a+b)(a-b)=a^2-b^2.}$

$\mathbf{a^2-b^2=(a+b)(a-b).}$

We may illustrate this graphically (fig. 6).



The whole figure $=a^2$. It is composed of an unshaded portion, b^2, and a shaded portion which may be arranged to form a rectangle $(a+b)(a-b)$.

Hence $(a+b)(a-b)=a^2-b^2$.

And $a^2-b^2=(a+b)(a-b)$.

This result may be stated thus:

The product of the sum and difference of two quantities equals the difference of the squares of the quantities.

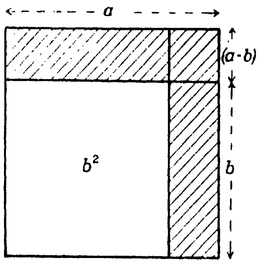


FIG. 6.

Example. Expand $(3x+7y)(3x-7y)$.

Expression $=(3x)^2-(7y)^2$.

$=9x^2-49y^2$.

EXERCISE 41

Expand the following:

1. $(a+b)(a-b)$; $(r-s)(r+s)$.
2. $(x+y)(x-y)$; $(2p-q)(2p+q)$.
3. $(ab+cd)(ab-cd)$; $(lm-rt)(rt+lm)$.
4. $(\frac{1}{2}a+2b)(\frac{1}{2}a-2b)$; $\left(\frac{b}{3}-\frac{c}{2}\right)\left(\frac{b}{3}+\frac{c}{2}\right)$.
5. $(R+r)(R-r)$; $(D-d)(D+d)$.
6. $\left(\frac{x}{y}+m\right)\left(m-\frac{x}{y}\right)$; $\left(\frac{2a}{3b}-\frac{c}{4d}\right)\left(\frac{2a}{3b}+\frac{c}{4d}\right)$.
7. $(\cdot 5c-\cdot 8d)(\cdot 5c+\cdot 8d)$; $(1\cdot 2x+1\cdot 1y)(1\cdot 2x-1\cdot 1y)$.
8. $(8a^2-b^2)(8a^2+b^2)$; $(\cdot 7x^2-y)(\cdot 7x^2+y)$.
9. $(x-a^2)(a^2+x)$; $(r^2-1)(r^2+1)$.

The expansion of the product of the sum and difference of two terms is in general $(a+b)(a-b)=a^2-b^2$.

Conversely: the difference of the squares of two quantities may be factorised as the product of their sum and difference.

Example. Resolve into factors $64y^2-81z^2$.

Expression $=(8y)^2-(9z)^2$.

$=(8y+9z)(8y-9z)$.

EXERCISE 42

Factorise the following :

1. p^2-q^2 ; s^2-t^2; $9x^2-81y^2$.
2. A^2-B^2 ; L^2-B^2 ; B^2-H^2.
3. $c^2-m^2n^2$; $49-a^2$; $\frac{1}{4}-k^2c^2$.
4. l^2-36p^2 ; $x^2-\frac{m^2}{9}$; $g^2-\frac{9}{4}h^2$.
5. $a^2+2ab+b^2-d^2$; a^2-25b^2.
6. $81x^2-36y^2$; $25a^2-16b^2$; p^2-9q^2.
7. $36m^2-n^2$; $121r^2-4t^2$; $49a^2-49b^2$.
8. $169a^2-b^2$; $r^2-\frac{4}{x^2}$; a^2-121m^2.
9. $4x^2-16y^2$; $9m^2-81n^2$; $36x^2-4z^2$.
10. $81a^4-36b^4$; $49x^2y^2-z^2$; a^2-81l^2.

Example. Find the area of the shaded portion of fig. 7, *which represents a washer or a flat ring.*

This area = Area of larger circle − Area of smaller circle.

$=\pi\mathbf{R}^2-\pi\mathbf{r}^2$.

$=\pi(\mathbf{R}^2-\mathbf{r}^2)$.

$=\pi(\mathbf{R}+\mathbf{r})(\mathbf{R}-\mathbf{r})$.

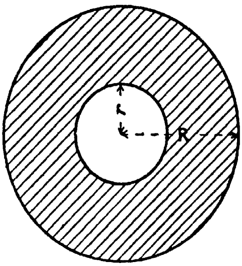


FIG. 7.

11. If in fig. 7 $R=3\cdot5$ ins. and $r=2\cdot4$ ins., find the area of the shaded portion. ($\pi=3\cdot14$.)

12. Find the upper surface area of a washer having an outer diameter of $2\frac{1}{2}$ ins. and an inner diameter of 2 ins. ($\pi=3\cdot14$.)

13. If fig. 7 represents a circular lawn with a path round it, what will be the cost of asphalting the path at 3s. 4d. per square yard, if the lawn has a diameter of 18 yds., and the path is 6 ft. in width ? ($\pi=3\frac{1}{7}$.)

14. A circular tower x ft. thick and y ft. high has an outer

diameter of $28x$ ft. Find, in cubic feet, the volume of the building material.

15. A gasholder has a diameter of $8b$ ft., and is surrounded by a path $8b$ ins. wide. Write an expression for the area of the path in square feet.

Exercise 43

1. Show by a diagram that $k(a+b+c)=ka+kb+kc$.

2. A shopkeeper serves four customers with a lbs., b lbs., c lbs., and d lbs. of tea respectively. If the tea costs x pence per lb., write an expression to show in shillings the total value of the tea sold.

3. A farmer rents a rectangular field m yds. by n yds., and his neighbour's field is p yds. longer and q yds. broader. Find the difference in the area of the two fields.

4. A room is x yds. by y yds., and the carpet in the room is a yds. less in length and b yds. less in breadth. Find the area of the uncarpeted floor.

5. Find the area of the picture frame represented in fig. 8.

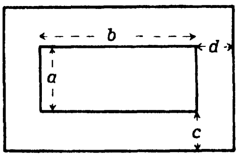


Fig. 8.

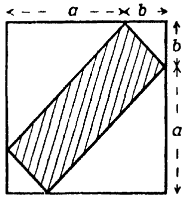


Fig. 9.

6. Show by a diagram that $(a+b)^2=a^2+b^2+2ab$.

7. Find the area of the shaded rectangle in fig. 9.

8. The length of a square playground was originally p yds. The side was increased by t yds. and the playground again made into a square. By how much was the area thus increased ?

9. Show by a diagram that $(a-b)^2=a^2+b^2-2ab$.

10. If the area of a square room is 196 sq. ft., and a square carpet in the room is $4m$ ft. shorter than the side of the room, find the area of the carpet.

11. Find the area of the unshaded portion of fig. 10.

12. Show by a diagram that $a^2-b^2=(a+b)(a-b)$.

13. Fig. 11 shows a square blotting pad. Give a one-term expression for the area of the blotting paper exposed.

14. The surface area of the top of a metal washer is given by

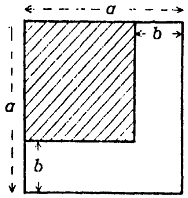


FIG. 10.

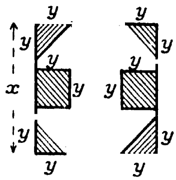


FIG. 11.

the formula $\pi(R^2-r^2)$ when R and r are the external and internal radii respectively. Show how this area may be calculated without squaring the lengths of the radii.

15. The volume of the material in a hollow cylindrical pipe equals the surface area of the base multiplied by the length

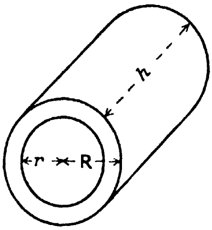


FIG. 12.

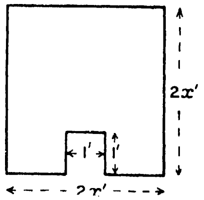


FIG. 13.

of the pipe. Find a one-term expression for the volume of the material of the pipe shown in fig. 12.

16. Fig. 13 shows the end elevation of a hen coop. Find the cost of covering this end with felt at 3d. per square foot.

17. The triangular plate shown in fig. 14 weighs x ozs. per square inch. Write a one-term expression for the total weight of two such plates.

18. Find the area remaining when the shaded portion is cut away from the quadrant (fig. 15).

19. What area remains when b slots, each 4 ins. by b ins., are punched from a plate 16 ins. by 16 ins. ?

20. After 16 circular holes, each having a radius p ins., are

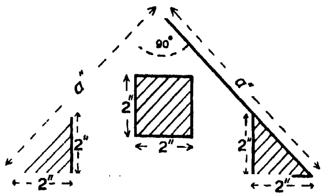


FIG 14.

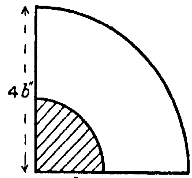


FIG. 15.

punched out of a circular board of radius r ins., what area of wood remains ?

Simplification

By means of factors, complicated expressions may often be simplified.

Example. *Simplify* $\dfrac{x^2-y^2}{x^2-2xy+y^2}\times\dfrac{x^2-y^2}{x+y}$.

$$\text{Expression}=\frac{(x+y)(x-y)}{(x-y)(x-y)}\times\frac{(x+y)(x-y)}{x+y}.$$
$$=x+y.$$

EXERCISE 44

Simplify:

1. $\dfrac{4x^2-4y^2}{x^2+2xy+y^2}$; $\dfrac{a^2-5ab-24b^2}{a^2-16ab+64b^2}$.
2. $\dfrac{ma^2+2mab+mb^2}{a^2-b^2}$; $\dfrac{x^2-14x+49}{x^2-2x-35}$.
3. $\dfrac{2x-9}{4x^2-36x+81}$; $\dfrac{625x^4-y^4}{25x^2+y^2}$.
4. $\dfrac{12+b}{b^2+18b+72}$; $\dfrac{a^2-10a+16}{a^2-64}$.
5. $\dfrac{a^2-b^2}{x-y}\times\dfrac{x^2-2xy+y^2}{a+b}$; $\dfrac{p^2+8p+12}{p^2+12p+36}$.
6. $\dfrac{7a-2b}{7a+2b}\times\dfrac{49a^2-4b^2}{49a^2-28ab+4b^2}$; $\dfrac{m^2-5m-36}{m-9}\times\dfrac{2x}{m+4}$.

7. $\dfrac{a^2-1}{a+1} \times \dfrac{1}{a-1}$; $\dfrac{25a-5b}{25a^2-b^2}$.

8. $\dfrac{a}{x+a} + \dfrac{ax+x^2}{(x+a)^2}$; $\dfrac{2a^2+4ab+2b^2}{a^2-b^2}$.

9. $\dfrac{x-y}{x+y} + \dfrac{x+y}{x-y}$; $\dfrac{x^2+2x-35}{x+7} \times \dfrac{x^2+4x+3}{x+3}$.

10. $\dfrac{a+b}{a-b} - \dfrac{a-b}{a+b}$; $\dfrac{1}{a+b} \times \dfrac{a^2-b^2}{a^2-2ab+b^2}$.

Square Root

Since $+a \times +a = a^2$ and also $-a \times -a = a^2$, it follows that

$$\sqrt{a^2} = +a \text{ or } -a.$$

For the present we shall consider only the positive root. The square root of a single term can often be found by inspection.

Example.

$$\sqrt{a^2b^4} = ab^2$$
$$\sqrt{16x^4} = 4x^2$$
$$\sqrt{\frac{64a^4}{9x^2}} = \frac{8a^2}{3x}.$$

Exercise 45

Find by inspection the positive square roots of:

1. a^4.
2. a^6b^4.
3. $81x^4y^2$.
4. $16a^2b^6$.
5. $49x^2y^4z^8$.
6. x^6y^8.
7. $\cdot01a^4b^4$.
8. $\cdot16x^2y^4$.
9. $13\frac{4}{9}x^4$.
10. $\cdot0064a^2m^6$.
11. $\frac{9}{64}a^{12}b^{16}$.
12. $\dfrac{16a^4}{9b^2}$.
13. $\dfrac{a^6b^4}{\cdot0001}$.
14. $1\cdot21m^6n^{10}$.
15. $\dfrac{81a^4b^2}{c^8}$.
16. $\dfrac{\cdot16x^4}{\cdot04a^2}$.
17. $\frac{16}{49}a^8b^{10}$.
18. $(2abc^2)^2$.
19. $\cdot09(3ab)^2$.
20. $16(xy^2z)^4$.

As shown above, the square root of a single term is always a single term.

A binomial expression cannot have a square root.

A trinomial expression (*i.e.* an expression of three terms), when a perfect square, must be in the form

$$a^2+2ab+b^2 \text{ or } a^2-2ab+b^2.$$

Since (i) $a^2+2ab+b^2=(a+b)^2$.

(ii) $a^2-2ab+b^2=(a-b)^2$.

It follows that (i) $\sqrt{a^2+2ab+b^2}=(a+b)$

(ii) $\sqrt{a^2-2ab+b^2}=(a-b)$.

The square root of any trinomial of this type may be found by inspection.

Example. $\sqrt{16p^4-40p^2m+25m^2}=4p^2-5m$.

EXERCISE 46

Write down the square roots of:

1. $16x^2+24xy+9y^2$.
2. $9x^4+12x^3y+4x^2y^2$.
3. $1+6x^3+9x^6$.
4. $\cdot 25b^2+4b+16$.
5. $a^4-\dfrac{a^2}{2}+\cdot 0625$.
6. $(x+y)^2+2(x+y)+1$.
7. $(x-y)^2-4(x-y)+4$.
8. $a^2-\dfrac{4ab}{5}+\dfrac{4}{25}b^2$.
9. $1-14x+49x^2$.
10. $a^2b^2-8ab+16$.

Consider the following method of finding the square root:

Example. Find the value of $\sqrt{a^4+4a^3+6a^2+4a+1}$.

$$
\begin{array}{r|l}
a^2 & a^4+4a^3+6a^2+4a+1\,(a^2+2a+1 \\
 & a^4 \\
\hline
2a^2+2a & \quad +4a^3+6a^2+4a+1 \\
 & \quad +4a^3+4a^2 \\
\hline
2a^2+4a+1 & \qquad\quad 2a^2+4a+1 \\
 & \qquad\quad 2a^2+4a+1 \\
\hline
 & \qquad\quad \cdot \quad \cdot \quad \cdot
\end{array}
$$

$\therefore \sqrt{a^4+4a^3+6a^2+4a+1}=a^2+2a+1$.

Steps

1. Arrange the expression in descending powers of a.
2. Find the square root of a^4, that is, the first term; it is a^2. Write a^2 as the first term of the required root.

3. Square a^2 and subtract the result from the first term.
4. Bring down the rest of the expression to form the new dividend.
5. Multiply a^2 by 2 and set down $2a^2$ as the first term of the new divisor.
6. Divide $4a^3$, the first term of the new dividend, by $2a^2$, the first term of the new divisor, and add the result, $2a$, to the root, and also to the new divisor.
7. Multiply the whole divisor $2a^2+2a$ by $2a$, and subtract the result from the dividend.
8. Continue similarly to the end.

Exercise 47

Find the square root of:

1. $625a^2+50a+1$.
2. $4x^2-72x+324$.
3. $9a^4-12a^2b+4b^2$.
4. $49x^4+28x^2y+4y^2$.
5. $a^2+b^2+c^2+2ab+2bc+2ac$.
6. $9x^2+4y^2+12xy+4y+6x+1$.
7. $p^2+4q^2+4pq+8q+4p+4$.
8. $4a^4+4a^3+5a^2+2a+1$.
9. $9x^4+6x^3-11x^2-4x+4$.
10. $4x^4+4x^3+5x^2+2x+1$.

Easy Simultaneous Equations

Consider the following statements: (i) A dealer bought coal and oil which together cost £44; (ii) the coal cost £3 more than the oil.

If x = the value of the coal in £'s, and y = the value of the oil in £'s, the statements may be expressed as equations, thus:

$$\text{(i)}\quad x+y=44$$
$$\text{(ii)}\quad x-y=3$$

There is an unlimited number of values of x and y which satisfy equation (i), and also an unlimited number of values of x and y which satisfy equation (ii).

If, however, equation (i) is true **at the same time** as equation (ii), then x and y must have the values $x=23\frac{1}{2}$, $y=20\frac{1}{2}$, and these values only.

When two or more equations are satisfied by the *same* values of the unknowns, they are called **Simultaneous Equations.**

They may be solved (i) by elimination, or (ii) by substitution.

Solution—by Elimination

Example i. *Solve* $4x+3y=17$ . . . Equation (i)
$3x+6y=24$ . . . „ (ii)

Multiply equation (i) by 3 and equation (ii) by 4.

We get $12x+9y=51$ . . . „ (iii)
$12x+24y=96$ . . . „ (iv)

Subtract equation (iii) from equation (iv) and we get

$$15y=45$$
$$\therefore \quad y=3$$

Now substitute this value of y in equation (i), and we get

$4x+9=17$ or in equation (ii), and we get $3x+18=24$
$\therefore\ 4x=8$ $\therefore\ 3x=6$
$\therefore\ x=2$ $\therefore\ x=2$

Solution is $x=2$ and $y=3$. (Check this.)

Be on the lookout for the following points when working Ex. 48:

(i) We may need to multiply one equation only, as in No. 1.

(ii) The values of one of the unknowns may be made equal by division, as in No. 2.

(iii) The elimination may be done by addition, as in No. 2.

(iv) No multiplication may be necessary, as in No. 4.

Solution—by Substitution

Example ii. *Solve* $x-2y=0$ . . . Equation (i)
$3x+4y=30$ . . . „ (ii)

From equation (i) $x=2y$.

Substitute this value of x in equation (ii), and we get

$$3(2y)+4y=30$$
$$\therefore \quad 10y=30$$
$$\therefore \quad y=3$$

Now substitute this value of y in equation (i), and we get

$x-6=0$ or in equation (ii), and we get $3x+12=30$
$\therefore\ x=6$ $\therefore\ 3x=18$
$\therefore\ x=6$

Solution is $x=6$ and $y=3$. (Check this.)

(i) When necessary, transpose terms (Ex. 49, No. 3).

(ii) When fractions occur, convert them to whole numbers by multiplying both sides of the equation by the L.C.M. of their denominators (Ex. 49, No. 7).

EXERCISE 48

Solve (using the method of elimination):

1. $3x-4y=1.$
$x+2y=7.$

2. $10x+15y=40.$
$2x-2y=-2.$

3. $7x+3y=26.$
$3x+7y=34.$

4. $x-2y=1.$
$x+3y=16.$

5. $6x+16=11y.$
$15x+4y=23.$

6. $\frac{x}{2}+\frac{y}{2}=6.$
$x-y=4.$

7. $\frac{x}{3}+\frac{a}{2}=3.$
$\frac{x}{2}+\frac{a}{4}=2\frac{1}{2}.$

8. In 7 hrs. A walks 13 miles more than B walks in 2 hrs. In 3 hrs. B walks 6 miles more than A walks in 2 hrs. Find the rate of each in miles per hour.

9. The sum of the two base angles of a certain triangle is 100°. Half their difference is 10°. Find the magnitude of each.

10. Two lbs. of tea and 1 lb. of sugar cost 5s. 9d.; 1 lb. of tea and 6 lbs. of sugar cost 5s. 2d. Find the price per lb. of each.

EXERCISE 49

Solve (by the method of substitution):

1. $12x=4y.$
$2x-y=-10.$

2. $a-4b=0.$
$7b+3=2a.$

3. $p-4=q.$
$3q-1=2p.$

4. $4a-2b=8.$
$7a+3b=27.$

5. $a=5b.$
$8b+1=a+7.$

6. $4x-3y=5.$
$9x=3y.$

7. $\frac{a}{3}+\frac{b}{2}=6.$
$8b-3a=-4.$

8. Find two numbers such that three times the larger exceeds five times the smaller by 11, and twice the larger equals seven times the smaller.

9. A motor car travels for 2 hrs. at one speed and for 4 hrs. at another speed. It travels altogether 220 miles. If it had travelled for 6 hrs. at the first speed and for 2 hrs. at the

second speed, it would have travelled 360 miles. Find the two speeds.

10. If $A=6rt$ and $V=9r^2t$, find t and r when $A=90$ and $V=270$.

EXERCISE 50

Solve by either method :

1. $a+b=48.$
$a-b=26.$

2. $2x+y=72.$
$x-y=51.$

3. $\frac{2x}{7}-\frac{y}{2}=120.$
$\frac{12x}{7}+\frac{y}{2}=410.$

4. $18x-3y=24.$
$4x+2y=0.$

5. $7a-3b=44.$
$2a-4b=0.$

6. $r^2+m=148.$
$r^2-m=94.$

7. $\frac{x}{2}-2=\frac{y}{3}.$
$\frac{x}{4}-\frac{5}{2}=\frac{2y}{3}.$

8. $12a=9b.$
$6b-18=5a.$

9. $m=4p.$
$4m=4(7p+3).$

10. $2a-9=5b.$
$3a-13=8b.$

11. The sum of two numbers is 29 and their difference is 6. Find the numbers.

12. A cup of tea and a biscuit cost $5\frac{1}{2}$d. ; three cups of tea and four biscuits cost 1s. 6d. Find the cost of two cups of tea and three biscuits.

13. A box of handkerchiefs costs half a crown. The handkerchiefs cost a florin more than the box. What was the cost of the handkerchiefs ?

14. A boy and his sister each receive an equal sum of money from their father. The boy trades with his money and doubles it, but his sister loses 5s. of her money. The mother now gives an equal sum of money to each, and the boy has then 36s. and his sister has 14s. Find how much money each parent gave.

15. A and B start out on a journey. For the first stage each pays an equal sum of money. The second stage costs B twice as much as A. If they pay respectively 2s. and 2s. 6d. for the total journey, what did each pay for the second stage ?

16. A fruit grower has apple trees and pear trees in his

orchard. Altogether he has 120 trees. If he had three times as many apple trees, he would have 266 trees altogether. How many of each has he ?

17. A number consists of two figures whose sum is 9. The value of the first figure is 68 more than that of the second figure. Find the number.

18. Each of two gardeners owns a similar square plot of ground. The first adds b sq. yds. to his plot and the second adds twice as much to his. The first has now 93 sq. yds., and the second 105 sq. yds. Find the length of each gardener's plot at first.

19. A man takes a journey by cycle and motor car. He cycles at 12 miles per hour and motors at twice that rate. In 10 hrs. he has travelled 204 miles. How long did he cycle ?

Graphs

Values of the same kind may be compared by lines drawn in the same direction. If only one line be drawn and then divided into equal divisions, such a line is spoken of as an **axis**, and values may be represented by various lengths measured from one end of it.

Draw on squared paper a **horizontal** line 48 units long, and then divide it into 12 equal parts. If each part represents 1 year, indicate lengths representing 3 yrs., 5 yrs., 8 yrs., 12 yrs.

Again, draw a **vertical** line 60 units in length and divide it into 5 equal parts. If each part represents a foot, indicate heights to represent 3 ft., 3 ft. 4 ins., 3 ft. 10 ins., 4 ft. 6 ins.

When it is required to compare two values which are not of the same kind, *e.g.* a boy's age and his height, two axes are used. The axes are arranged at right angles and their common point is called the **origin**.

Fig. 16 shows graphically the following data :

Boy's age .	3 yrs.	5 yrs.	8 yrs.	12 yrs.
„ height	3 ft.	3 ft. 4 ins.	3 ft. 10 ins.	4 ft. 6 ins.

The positions of the points A, B, C, and D indicate the boy's heights at 3, 5, 8, and 12 yrs. respectively. The line joining A, B, C, and D is known as the **curve**, and it shows how the boy's height varies with his age.

Note

(i) The scale used should be as large as possible.
(ii) The name of each value compared should be written by the side of its axis.

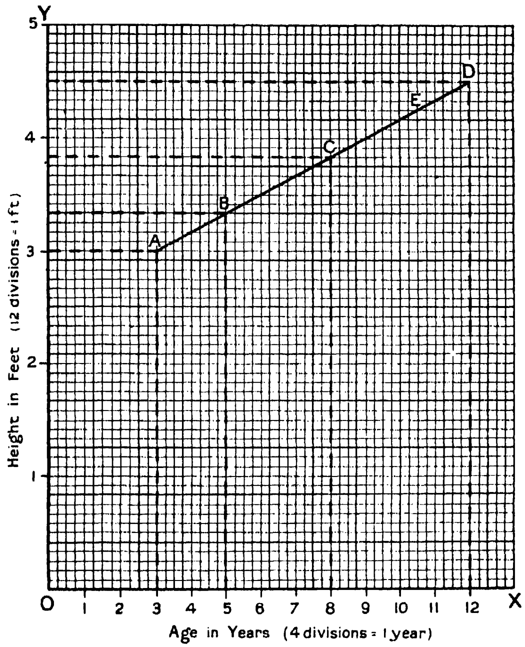


FIG. 16.

(iii) There should be a separate scale for each set of variables, and this scale should be stated.
(iv) The axes are lettered OX and OY for purposes of reference.
(v) The position of every point along AD represents the boy's height at a certain age, *e.g.* the position of E shows that the boy was 4 ft. 3 ins. in height at 10 yrs. 6 months, assuming that his growth was uniform.

Example. Show graphically on squared paper the cost of coal at 60*s. per ton, and from the graph find the cost of* 14 *cwts. and* 10 *cwts. of coal.*

Steps

(i) Draw axes OX and OY and label them respectively " Weight in cwts." and " Cost in shillings " (fig. 17).
(ii) Decide the scales.
(iii) Since no tons at 60s. per ton cost nothing, the curve must pass through O.

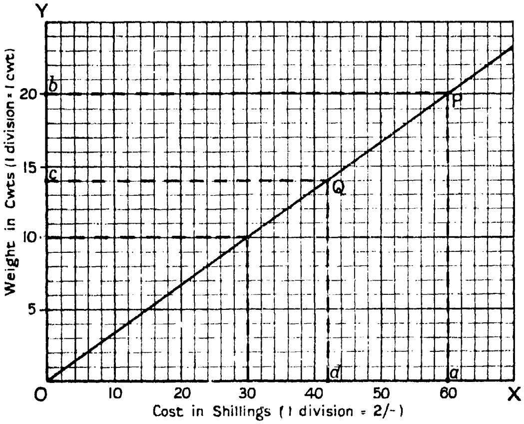


Fig. 17.

(iv) Find the length Oa along OX to represent 60s., and the length Ob along OY to represent 20 cwts.
(v) At a and b erect perpendiculars to meet at P.
(vi) Join OP and produce it if required. OP is the curve required.
(vii) Find the length Oc, along OY, to represent 14 cwts., and at c erect a perpendicular to meet OP in Q.
(viii) Draw Qd perpendicular to OX; then length Od represents the required value, namely 42s.
(ix) Find the value of 10 cwts., as shown.
(x) Check results arithmetically.

Exercise 51

1. Using squared paper, find the value of x in the following:

$$\text{(i) } \frac{x}{8}=\frac{5}{20}; \quad \text{(ii) } \frac{7}{x}=\frac{2}{6}; \quad \text{(iii) } \frac{4}{9}=\frac{x}{72}.$$

Check the results arithmetically.

2. Construct a graph to show the relation of pounds sterling to francs, assuming that £1=25 francs. From the graph

find the values of 10s., 14s., and £1, 2s. 0d. in francs, and the values of 20, 15, and 30 francs in shillings.

3. The barometer in a school was read daily at noon, and the following observations were made :—

Day.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
Height of Barometer	30·2	31·0	30·8	30·8	31·5	32·0

Plot a graph, and from it find the probable height of the barometer (*a*) at Wednesday midnight; (*b*) at Friday midnight.

4. Construct a graph so that we may find the value of pints in litres and *vice versa*, having given that $1\frac{3}{4}$ pints=1 litre. From the graph find the value of $4\frac{1}{2}$ pints in litres and the value of 5·5 litres in pints.

5. Draw a graph to show the simple interest on sums of money between £1 and £50 at 5 per cent. per annum.

6. Plot the following time-table on squared paper :

Distance.		Time.
. .	Manchester.	2 p.m.
61 miles .	Derby.	3·30 p.m.
90 ,,	Leicester.	4·10 ,,
190 ,,	London.	6·10 ,,

From the graph find the time when the train is halfway between Manchester and London.

7. In raising objects by means of a lever the results were as follows :

Effort (in lbs.)	20	35	40	50	65	70
Load (in lbs.) .	140	245	280	350	455	490

Plot a graph to illustrate these results. What load would be raised by an effort of 56 lbs. ?

8. Draw a graph from which to obtain the distance travelled by a train at any time during the 2 hrs. after starting, if the speed is uniformly 30 m.p.h. Using the graph, find (*a*) the distance travelled in 40 mins.; (*b*) the time taken to travel 38 miles.

9. If 4 sq. ft. of copper sheeting of uniform thickness weigh 12 lbs., find graphically the weight of 9 sq. ft., $8\frac{1}{2}$ sq. ft., $6\frac{1}{2}$ sq. ft. of such sheets, and also the area of similar copper sheets weighing 10 lbs., 14 lbs., and 20 lbs.

10. Booklets are sold at 15s. per 100. Draw a graph from which may be read the prices of any number of fifties up to 600. From the graph find the price of each fifty. Compare the results with those obtained by calculation.

11. Show by means of a graph the relation between degrees Fahrenheit and degrees Centigrade, having given that 0° C.=32° F. and 100° C.=212° F. From the graph find the values of 50° F. and 70° F. in degrees Centigrade and of 30° C. and 40° C. in degrees Fahrenheit.

Indices

The number which indicates how many equal factors make up a product is known as the **index**; the product is known as the **power**; the factor is known as the **root** or **base** of the power.

Thus, $7^3=343$. In this case the index is 3, the power 343, and the root or base is 7.

When we raise a number to a given power, the process is known as *involution*. The reverse process, *i.e.* finding the root, base, or factor of a power, is known as *evolution*.

The root of a power is indicated thus:

$\sqrt{4}=2$, *i.e.* the second or square root of $4=2$.

$\sqrt[3]{27}=3$, *i.e.* the third or cube root of $27=3$.

$\sqrt[4]{b^4}=b$, *i.e.* the fourth root of $b^4=b$.

The Laws of Indices

I.

(i) $$\begin{aligned} 2^2\times 2^3 &= (2\times 2)\times(2\times 2\times 2)\\ &=2\times 2\times 2\times 2\times 2\\ &=2^5 \text{ or } 2^{2+3}. \end{aligned}$$

(ii) $$\begin{aligned} a\times a^3 &= a\times(a\times a\times a)\\ &=a\times a^3\\ &=a^4 \text{ or } a^{1+3}. \end{aligned}$$

These examples illustrate the *First Index Law*. ***The product***

of two or more powers of the same quantity has an index equal to the sum of the indices.

II. (i) $2^3 \div 2^2 = \dfrac{2\times 2\times 2}{2\times 2}$.

$= 2$ or 2^1 or 2^{3-2}.

(ii) $a^3 \div a = \dfrac{a\times a\times a}{a}$

$= a^2$ or a^{3-1}.

These examples illustrate the *Second Index Law. The quotient of two powers of the same quantity has an index equal to the difference of the indices of the dividend and the divisor.*

Note two special cases:

(i) $$6^2 \div 6^5 = \frac{6\times 6}{6\times 6\times 6\times 6\times 6} = \frac{1}{6\times 6\times 6} = \frac{1}{6^3}.$$

By the Second Index Law $6^2 \div 6^5 = 6^{2-5} = 6^{-3}$.

$$\therefore \frac{1}{6^3} = 6^{-3}.$$

Similarly $$\frac{1}{4^2} = 4^{-2};\quad \frac{1}{a^2} = a^{-2};$$

$$\frac{1}{10^2} \text{ or } \frac{1}{100} \text{ or } \cdot 01 = 10^{-2}.$$

(ii) $$6^3 \div 6^3 = \frac{6\times 6\times 6}{6\times 6\times 6} = 1.$$

By the Second Index Law $6^3 \div 6^3 = 6^{3-3} = 6^0$.

$$\therefore 6^0 = 1.$$

Similarly $a^0 = 1$; $b^0 = 1$, etc.

Any power having the index $0 = 1$.

III. (i) $(2^2)^3 = (2\times 2)\times(2\times 2)\times(2\times 2) = 2^6$ or $2^{2\times 3}$

$(4\times 3)^2 = 4^2\times 3^2$

$(4^3\times 3^2)^2 = (4^3)^2\times(3^2)^2 = 4^6\times 3^4$.

(ii) $(x\times y)^3 = x^3\times y^3 = x^3y^3$.

(iii) $(x\times y)^n = x^n\times y^n = x^ny^n$.

Hence we have the *Third Index Law. To raise a product to a power, raise each factor to the same power.*

The Three Laws may be stated thus:

First Index Law . . .		$x^a\times x^b = x^{a+b}$
Second Index Law . . .	(i)	$x^a \div x^b = x^{a-b}$
	(ii)	$x^{-a} = \dfrac{1}{x^a}$.
	(iii)	$x^0 = 1$.
Third Index Law . . .	(i)	$(x^a)^b = x^{ab}$.
	(ii)	$(x\times y)^n = x^n\times y^n$.

These laws are true for fractional values also.

Thus
$$x^{\frac{2}{3}}\times x^{\frac{2}{3}}=x^{\frac{4}{3}}.$$
$$x^{\frac{2}{3}}\div x^{\frac{1}{4}}=x^{\frac{2}{3}-\frac{1}{4}}=x^{\frac{5}{12}}.$$
$$(x^{\frac{2}{3}})^{\frac{1}{4}}=x^{\frac{2}{3}\times\frac{1}{4}}=x^{\frac{1}{6}}.$$

Since
$$x^{\frac{1}{2}}\times x^{\frac{1}{2}}=x \quad \therefore \sqrt{x}=x^{\frac{1}{2}}.$$

Similarly $\sqrt[3]{x}=x^{\frac{1}{3}}$; $\sqrt[4]{x}=x^{\frac{1}{4}}$, and $\sqrt[5]{x^3}=x^{\frac{3}{5}}$.

Exercise 52

1. Express the following products as powers of one quantity :
$x^2\times x^3$; $a^3\times a^2$; $b\times b^4$; $6^3\times 6^2$; $8^3\times 8^2$;
$a\times a^2\times a^3$; $x^3\times x^5\times x$; $m^x\times m^x$; $2^3\times 2^2\times 2^6\times 2$.

2. Express the following quotients as powers of one quantity :
$a^3\div a^2$; $x^7\div x^5$; $b^4\div b^2$; $7^3\div 7$; $4^3\div 4^2$;
$\frac{6^2}{6}$; $\frac{5^7}{5^3}$; $\frac{10^4}{10^2}$; $\frac{6^{\cdot 5}}{6^{\cdot 25}}$; $\frac{x^p}{x^q}$.

3. Simplify the following:
$(a^6)^2$; $(x^3y^3)^2$; $(m^n)^p$; $(10^4)^3$; $(6^{\frac{1}{2}})^4$; $(3^2)^{\frac{1}{2}}$.

4. Evaluate the following :
(i) 4^2 ; 8^3 ; 9^1 ; 10^3 ; 5^0.
(ii) 4^{-2} ; 8^{-3} ; 9^{-1} ; 10^{-3}. (State results as fractions.)
(iii) $144^{\frac{1}{2}}$; $512^{\frac{1}{3}}$; $\sqrt[3]{10^6}$; $4^{\frac{4}{2}}$; $(ab^2c)^0$; $(234\frac{1}{2})^0$.

5. Express the following as powers of one quantity :
(i) $a^3\times a^3\div a^2$; $x^4\times x^2\times x\div x^3$; $\frac{a^6\times a^2}{a^3}$; $\frac{m^x\times m^y}{m^z}$.
(ii) $7^2\times 7^2\div 7^4$; $6^{\frac{1}{2}}\times 6^{\frac{1}{3}}\div 6^3$; $18^7\div 18^6\div 18$.
(iii) $9^{1\cdot 5}\times 9^{2\cdot 3}$; $8^{\frac{2}{3}}\div 8^{\frac{1}{4}}$; $12^2\times 12^3\div 12^4\div 12$.

6. Find the value of the following, giving answers to (ii) both in the fractional and in the decimal form :
(i) 10^2 ; 10^3 ; 10^5 ; 10^6 ; 10^0.
(ii) 10^{-2} ; 10^{-3} ; 10^{-5} ; 10^{-6} ; 10^{-7}.
(iii) $(10^2)^3$; $(10^2)^4$; $(10^3)^2$; $(10^4)^2$; $(10^{\frac{1}{2}})^4$.
(iv) $100^{\frac{1}{2}}$; $1000^{\frac{1}{3}}$; $10{,}000^{\frac{1}{4}}$.

Introduction to Logarithms

By means of logarithms (written briefly **logs**) we can perform multiplication by addition, division by subtraction, involution by multiplication, and evolution by division.

A logarithm is the index of a power to which an invariable number, called the **base**, has to be raised in order to produce the number of which it is the logarithm.

Consider this table:

$$2^1 = 2$$
$$2^2 = 4$$
$$2^3 = 8$$
$$2^4 = 16$$

Speaking in terms of indices we say:

1 is the index of the power 2 to the base 2.
2 ,, ,, ,, ,, 4 ,, ,, 2
3 ,, ,, ,, ,, 8 ,, ,, 2
4 ,, ,, ,, ,, 16 ,, ,, 2

Speaking in terms of logs we say:

1 is the log of 2 to the base 2 or $\log_2 2 = 1$
2 ,, ,, 4 ,, ,, 2 or $\log_2 4 = 2$
3 ,, ,, 8 ,, ,, 2 or $\log_2 8 = 3$
4 ,, ,, 16 ,, ,, 2 or $\log_2 16 = 4$

Consider the following:

I. $10^1 = 10$ that is $\log_{10} 10 = 1$
$10^2 = 100$,, $\log_{10} 100 = 2$
$10^3 = 1000$,, $\log_{10} 1000 = 3$
$10^4 = 10{,}000$,, $\log_{10} 10{,}000 = 4$

As the base 10 is the one generally used, logarithms to the base 10 are known as **common logarithms**, and the figure 10 is omitted. Thus log 1000 = 3 means "the log of 1000 to the base 10 = 3."

II. $10^1 = 10$ that is $\log 10 = 1$
$10^0 = 1$,, $\log 1 = 0$
$10^{-1} = \frac{1}{10} = \cdot 1$,, $\log \cdot 1 = -1$
$10^{-2} = \frac{1}{100} = \cdot 01$,, $\log \cdot 01 = -2$

III. $10^{\frac{1}{2}}$ or $10^{\cdot 5} = 3{\cdot}1623$ that is $\log 3{\cdot}1623 = \cdot 5$
$10^{\frac{1}{4}}$,, $10^{\cdot 25} = 1{\cdot}7783$,, $\log 1{\cdot}7783 = \cdot 25$
$10^{\frac{1}{8}}$,, $10^{\cdot 125} = 1{\cdot}3335$,, $\log 1{\cdot}3335 = \cdot 125$

Example (i). *Find by logs the value of* $100 \times 100 \times 1000 \times 1$.

Log of answer = log 100 + log 100 + log 1000 + log 1
$= 2 + 2 + 3 + 0 = 7$.

Number of which 7 is the log = 10,000,000.

$\therefore$ Value = 10,000,000.

Example (ii). *Find by logs the value of* $\dfrac{10\times 100\times 10000}{1000}$.
Log of answer $=1+2+4-3=4$.
$\therefore$ Value $=10{,}000$.

Example (iii). *Find by logs the value of* $\sqrt{10{,}000}$.
Log of answer $=\frac{1}{2}(\log 10{,}000)=\frac{1}{2}\times 4=2$.
$\therefore$ Value $=100$.

Example (iv). *Find by logs the value of* 100^3.
Log of answer $=3$ (log 100) $=3\times 2=6$.
$\therefore$ Value $=1{,}000{,}000$.

Example (v). *If* $8{\cdot}37=10^{\cdot 9227}$ *write the logs of*
(i) $83{\cdot}7$; (ii) 837 ; (iii) $\cdot 0837$.

(i) $83{\cdot}7=10\times 8{\cdot}37=10\times 10^{\cdot 9227}=10^{1\cdot 9227}$
$\therefore$ log $83{\cdot}7=1{\cdot}9227$.

(ii) $837=100\times 8{\cdot}37=10^2\times 10^{\cdot 9227}=10^{2\cdot 9227}$
$\therefore$ log $837=2{\cdot}9227$.

(iii) $\cdot 837=\cdot 1\times 8{\cdot}37=10^{-1}\times 10^{\cdot 9227}=10^{\bar{1}\cdot 9227}$
$\therefore$ log $\cdot 837=\bar{1}{\cdot}9227$.

Exercise 53

1. Construct a table of powers of 3 up to the 8th power and use it to evaluate $\dfrac{9\times 3\times 81\times 729}{243}$.

2. What is the value of:

(i) $\log_2 64$; $\log_2 1024$; $\log_4 64$; $\log 10{,}000$?
(ii) $\log_9 81$; $\log_5 25$; $\log_{10} 100$; $\log_3 27$?
(iii) $\log_5 625$; $\log_3 81$; $\log_x x^3$; $\log_m m^8$?

3. Express, using logs:

(i) $4^2=16$; $7^2=49$; $10^3=1000$; $4^4=256$.
(ii) $3^m=x$; $8^r=h$; $\sqrt[5]{m}=20$; $10^x=451$.
(iii) $10^{2\cdot 8756}=751$; $10^{2\cdot 4409}=276$; $10^0=1$; $10^{1\cdot 6127}=41$.

4. Write logs to the base 2 of the following:
4, 8, $\frac{1}{16}$, 256, $\frac{1}{32}$.

5. If $\log_x 36=2$, find the value of x ; if $\log_x 125=3$, find the value of x.

6. Express, using indices:

(i) $\log_3 81=4$; $\log_2 64=5$; $\log 100=2$.
(ii) $\log_4 256=4$; $\log_x 85=2$; $\log n=x$.

7. Evaluate:

(i) $\log 10{,}000$; $\log_3 729$; $\log_{\cdot 5} \cdot 125$; $\log_{11} 1331$.
(ii) $8 \log_2 2$; $2 \log 100$; $\frac{1}{2} \log 10{,}000$; $\log_3 81 + \log_3 9$.

8. Work by logs, using a suitable base for each case:

(i) 512×128; (ii) $10{,}000{,}000 \div 1000$; (iii) $\sqrt{729}$;
(iv) $\dfrac{16^2}{\sqrt{256}}$; (v) $\dfrac{\sqrt{1000000}}{1000}$.

9. Between what two whole numbers are the following logs: $\log_5 70$; $\log 78$; $\log 826$; $\log 3856$; $\log_2 70$; $\log_2 86$; $\log_2 150$?

10. Having given that $2{\cdot}56 = 10^{\cdot 4082}$, express each of the following numbers as powers of 10 (*i.e.* find their common logs):

256; 25,600; 25·6; 2560.

SECTION III

GEOMETRY AND MENSURATION

Revision—Length

EXERCISE 54

1. If 21 yds. of twine may be wrapped exactly 10 times round a cylindrical box, what is the diameter of the box? ($\pi=3{\cdot}14$.) Give the answer correct to the hundredth of an inch.

2. The "surround" of a gasometer has a diameter of 18 yds., and is divided into 8 equal sections. What is the length of each section? ($\pi=3{\cdot}14$.)

3. Fifty yds. of wire are made into 32 hoops. Find in feet the diameter of each hoop. ($\pi=3{\cdot}14$.)

4. Explain what is meant when it is said that the ratio between the circumference and the diameter of a circle is *constant*.

5. Draw a line 3·4 ins. long; from one end cut off 1·6 ins.; measure the remainder; compare with the arithmetical difference.

6. Draw a line 2·2 ins.; bisect it geometrically; check the result.

7. Draw a line 10·8 cm. long; produce it 2·6 cm. to the right and 1·3 cm. to the left; find the total length; compare with the arithmetical sum.

8. Show on squared paper a point P, 3·8 miles to the east and 1·6 miles to the north of a fixed point O. Measure the distance OP. Work to a scale of 1 in.=1 mile.

Angles and Parallel Lines

EXERCISE 55

1. Construct the following angles geometrically: 90°; 45°; 60°; 30°; 15°; 75°.

2. Draw a line $3\frac{1}{2}$ ins. in length; bisect it geometrically; test the result.

3. Draw an angle of 80° by using the protractor; bisect it geometrically; test the result.

4. Draw line AB (fig. 18) 2·3 ins. long; bisect it at a point O; construct ∠ BOC 30° and produce CO to D. Find the magnitude of ∠'s AOD, AOC, DOB. What is the sum of the ∠'s AOC and COB? And the sum of the ∠'s AOD and DOB?

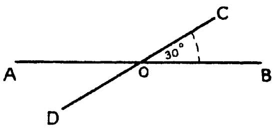


FIG. 18.

5. Draw two parallel lines AB and CD. Draw a line PQRS as shown in fig. 19, making ∠ PQB 60°. Name the vertically opposite angles, the alternate angles, and the corresponding angles formed, and state their magnitudes.

6. Draw a line AB $2\frac{1}{4}$ ins. long; bisect it at O; construct at O a perpendicular OC $2\frac{1}{4}$ ins. long, and produce CO to OD, making OD=OC. Name the four angles formed at the point O, and state their magnitude.

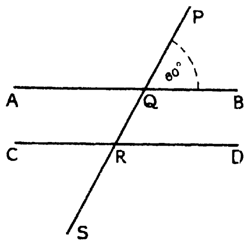


FIG. 19.

7. Show by drawings the angles made by the hands of a clock at 4 o'clock and at 10 o'clock. Note that there are two angles formed in each case. What is the sum of these two angles?

8. Construct geometrically a square on a line 1·7 in. long. Draw the diagonals cutting at a point O. State the magnitude of each angle formed.

Triangles

EXERCISE 56

1. Make drawings of (*a*) an equilateral triangle on a base of 1·6 in. and state the magnitude of each angle; (*b*) a right-angled isosceles triangle having two equal sides 1·8 ins. long and state the magnitude of each angle.

2. Place three points A, B, and C in such a position that each one is $1\frac{1}{2}$ ins. from the other two.

3. Construct on squared paper (*a*) any triangle having a base 2 ins. long and an altitude $1\frac{1}{2}$ ins.; (*b*) a rectangle on the same base and having the same altitude. Prove by

counting squares that the area of the triangle$=\frac{1}{2}$ the area of the rectangle.

4. Construct the triangle and rectangle of Question 3 above on plain paper, and prove by cutting and superposing that the area of the triangle$=\frac{1}{2}$ the area of the rectangle.

5. Construct a triangle on a base of $1\frac{3}{4}$ ins. equal in area to a rectangle $1\frac{7}{8}$ ins. $\times$ $1\frac{2}{5}$ ins. State the altitude of the triangle.

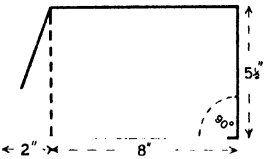


FIG. 20.

6. The area of a triangle is 26·88 sq. cm. If the base is 4·8 cm., find the altitude.

7. Fig. 20 represents a sheet of lead of uniform thickness. Find its weight if 1 sq. in. weighs 5 ozs.

8. A sheet of copper is in the form of a triangle. The base is 10·6 ft., the altitude is 9·8 ft., and it weighs 103·88 lbs. Find, in lbs., the weight of 1 sq. ft. of the copper.

Circles

EXERCISE 57

1. Find the area of one of the flat surfaces of a half-crown whose diameter is 1·2 ins. ($\pi=3{\cdot}14$.)

2. Find the cost, at 2s. 4d. per sq. ft., of a circular plate of zinc having a radius of 14 ins. ($\pi=3\frac{1}{7}$.)

3. Find the cost, at $3\frac{1}{2}$d. per sq. ft., of polishing the top of a pedestal which has a diameter of $2\frac{1}{2}$ ft. ($\pi=\frac{22}{7}$.)

4. Find the surface area of a circular lake whose greatest distance across is 22 yds. ($\pi=3\frac{1}{7}$.)

5. The diameter of the outer circle of stones at Stonehenge is 108 ft. Find, to the nearest square yard, the area of the surface within this circle. ($\pi=3{\cdot}14$.)

6. The diameter of a circle is found to be 17·5 ins., and its circumference 55·1 ins. Calculate the value of π to three decimal places.

7. A motor car has wheels of 3 ft. 6 ins. diameter. When they are making 120 revolutions per minute, find the speed of the car in feet per minute and in miles per hour. ($\pi=\frac{22}{7}$.)

8. A point on the rim of a circular saw is revolving at the rate of $\frac{1}{2}$ mile per minute. If the wheel is making 240 revolutions per minute, find its diameter. ($\pi=3\frac{1}{7}$).

Volume

EXERCISE 58

1. A rectangular prism is a ins. long, b ins. wide, and c ins. thick. Write (a) the total length of the edges; (b) the total area of the faces; (c) the volume.

2. Find the cubical content of (a) a cube with an edge of $3\frac{1}{2}$ ins.; (b) a cube with an edge of x ins.; (c) an oblong box with inside edges $3\frac{1}{2}$ ins. by $4\frac{1}{2}$ ins. by 16 ins.

3. The volume of a square prism is 5000 c.c., and its base contains 123 sq. cm. Find its altitude in centimetres to one place of decimals.

4. A block of wood measures 5·5 cm. × 4·5 cm. × ·5 cm. Find its cubical content to the nearest cubic centimetre.

5. Find the weight of a rectangular concrete flag measuring 24 ins. × 28 ins. × $2\frac{1}{2}$ ins. if concrete weighs 165 lbs. to the cubic foot.

6. The water in a full rectangular cistern 3 yds. long, 2 ft. wide, and 5 ft. high, is transferred to a larger empty tank 4 yds. long and 2 yds. wide. How high does the water rise in the larger tank?

7. Fig. 21 represents the end of an iron beam 10 yds. in length. Find its weight in lbs. if a cubic foot of iron weighs 480 lbs.

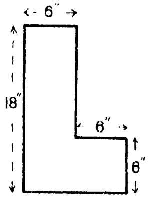


FIG. 21.

8. A cuboid, or rectangular prism, of lead measures 5 ins. × 2 ins. × $1\frac{1}{2}$ ins. Find its weight in lbs. if 1 cu. ft. of lead weighs 700 lbs.

Development

We have already considered the development of simple prisms such as the cube, rectangular prism, and cylinder.

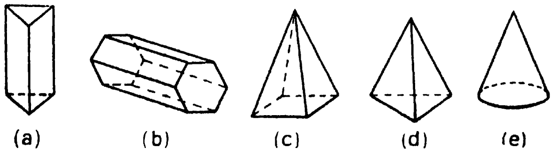


FIG. 22.

Consider fig 22 (a), (b), (c), (d), (e), representing (a) a triangular

prism (sometimes called a wedge), (*b*) a hexagonal prism, (*c*) a square pyramid, (*d*) a tetrahedron, and (*e*) a cone.

Since the total surface area of a solid is the sum of the areas of

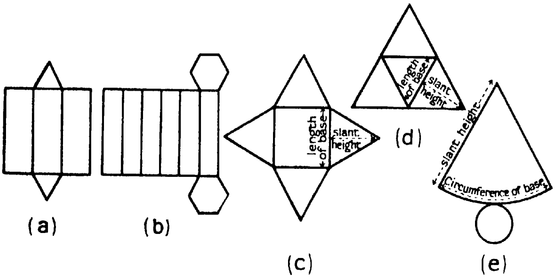


FIG. 23.

its separate faces, fig. 23 (*a*), (*b*), (*c*), (*d*), (*e*), represent the development of the solids shown on page 87.

Note

(i) The sides of all right prisms are rectangles.
(ii) The triangular sides of a pyramid are isosceles triangles having an altitude equal to the slant height of the pyramid.
(iii) The four faces of a tetrahedron are all equilateral triangles.

EXERCISE 59

1. Show the development of the walls of a room 20 ft. long, 15 ft. wide, and 10 ft. high. Scale 1 in. = 10 ft. Find the area of the walls.

2. A cylindrical vessel $1\frac{1}{2}$ ins. in height has an end diameter of $\frac{3}{4}$ in. Show its development.

3. Make a hand sketch of a triangular prism standing on an equilateral base $\frac{3}{4}$ in. in length, having an altitude of $1\frac{1}{2}$ ins. Draw the development of the prism and estimate its total surface area.

4. Examine a wedge, place it on its triangular base, and make a dimensioned hand sketch of it. Draw its development.

5. Draw on squared paper the development of a hexagonal prism. Cut the figure out, allowing for necessary flanges, and build up the prism. Make sketches of the model from three different points of view.

6. Construct a square on a base of 1·8 ins., and on each side of the square construct equilateral triangles having their vertices outside the square. Of what solid is your diagram the development ?

7. On the sides of an equilateral triangle draw three more equilateral triangles and cut out the resulting figure. Fold to form a solid. What is its name ? How many faces has it ? How many edges ? How many corners ?

8. Show the development of a conical tent having a base diameter of 10 ft. and a slant height of 13 ft. Scale 1 in. = 5 ft. ($\pi = 3·14$.)

Surface Area

EXERCISE 60

1. A solid cylinder of copper has an end whose radius is r ins., and its altitude is h ins.; write a formula for its total surface area. Find the total surface area when $r=1$ and $h=4$. ($\pi = 3·14$.)

2. A drain pipe has an outer diameter of 1 yd. and is 5 yds. in length. Find the area of its outer curved surface. ($\pi = 3\frac{1}{7}$.)

3. What weight of lead at 6 lbs. per square foot would be required to cover the inner curved surface of a cylindrical tank having an inner diameter of $3\frac{1}{2}$ ft. and a height of 6 ft. ? ($\pi = 3·14$.)

4. The diameter of the base of a cone is 22 ft., and its slant height is 10 ft. Find the area of the curved surface. ($\pi = \frac{22}{7}$.)

5. Find the cost of the canvas, at 3s. 6d. a square yard, required for a conical tent which is 6 yds. in diameter and whose slant height is 5 yds. ($\pi = \frac{22}{7}$.)

6. Find the total surface area of a square pyramid having one edge of its base $1\frac{1}{2}$ ins. and a slant height of 3 ins.

7. Find the cost, at 9d. per square yard, of painting the curved outer surface of a gasholder which is 15 yds. in height and which has a diameter of 15 yds. ($\pi = 3·14$.)

8. The surface area of a sphere $= 4\pi r^2 = 4\pi\left(\frac{d}{2}\right)^2 = 4 \times \pi \times \frac{d^2}{4}$ $= \pi d^2$.

Find the surface area of the following ($\pi = 3·14$) :

(*a*) A ball having a diameter of $3\frac{1}{2}$ ins.

(*b*) A marble having a diameter of $1\frac{1}{2}$ ins.

(*c*) A spherical football having a diameter of 1 ft.

(*d*) An orange having a diameter of 3 ins.

9. The area of a circle is sometimes said to be (diameter)^2 $\times 0{\cdot}7854$. Show how this formula may be obtained from πr^2, when $\pi = 3{\cdot}1416$.

10. Find the cost of covering a hemispherical dome with gilding, at 5s. a square inch, if the diameter is 1 yd. ($\pi = 3{\cdot}14$.)

11. Find the cost, at 2s. 6d. per sq. yd., of asphalting a path (6 ft. wide) round the outside of a rectangular plot 120 ft. by 90 ft.

12. The inside dimensions of an open rectangular tank are: length, 16 ft. 6 ins.; breadth, 14 ft. 6 ins.; height, 6 ft. 6 ins. Calculate the cost of painting the whole of the inside surface of the tank, at the rate of 1s. 9d. per sq. yd.

Useful Constructions

To divide a given straight line into any number of equal parts.

Let AB be the given straight line (fig. 24). At A construct

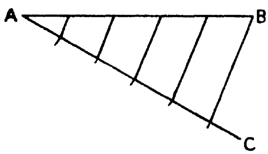


FIG. 24.

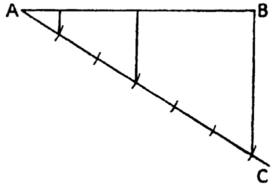


FIG. 25.

$\angle$BAC of any convenient size. Mark off the required number of equal divisions along AC. Join the last point to B and draw lines parallel to this line through all the other points. The divisions of AB are all equal.

To divide a straight line in a given ratio.

Let AB be the given straight line which is to be divided in the ratio 1 : 2 : 3 (fig. 25). Construct $\angle$BAC as before and mark off $1+2+3=6$ equal divisions along AC. Join the last point to B. Draw lines parallel to this line through the first and third points from A. The three divisions along AB are in the ratio 1 : 2 : 3.

Some Properties of Triangles and Parallelograms

(i)

Construct any scalene triangle ABC and label its sides as in fig. 26.

Measure ∠'s ABC, BCA, and CAB, and also measure sides *a*, *b*, and *c*.

Tabulate the results thus:

∠CAB=	°	side *a* =	ins.
∠ABC=	°	side *b* =	,,
∠BCA=	°	side *c* =	,,

Note.—(i) **The greatest angle is opposite the greatest side.**
(ii) **The smallest angle is opposite the shortest side.**

(ii)

Construct any triangle ABC and produce its sides in order

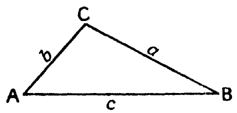


Fig. 26

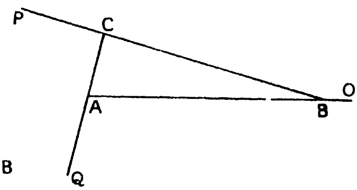


Fig. 27.

(fig. 27). Measure all the angles so formed and tabulate your results thus:

∠QAB =	∠ACB + ∠CBA =
∠OBC =	∠ACB + ∠CAB =
∠PCA =	∠CBA + ∠CAB =

Note.—**The exterior angle of a triangle is equal to the sum of the interior opposite angles.**

(iii)

Construct any parallelogram ABCD (fig. 28).

Produce DC in both directions.

On AB construct rectangle ABPQ as shown.

Then, area of parallelogram ABCD = AB × BP = area of rectangle ABPQ.

The area of a parallelogram is equal to that of a rectangle on the same base and between the same parallels.

(iv)

In fig. 28 draw also parallelogram ABMN as shown.

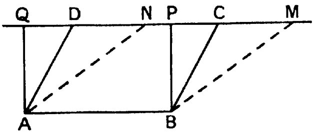


FIG. 28.

Area of parallelogram ABMN$=$AB$\times$BP$=$area of parallelogram ABCD.

The areas of parallelograms on the same base and between the same parallels are equal.

Since the diagonal of a parallelogram divides it into two equal triangles, it follows that:

The areas of triangles on the same base and between the same parallels are equal.

EXERCISE 61

1. Draw lines 3 ins., 3·5 ins., and 4·5 ins. in length, and divide them into 3, 7, and 9 equal parts respectively. Check by arithmetic.

2. Draw lines 27 mm., 3·6 cm., and 5·4 cm. in length, and divide them into 3, 9, and 6 parts respectively. Compare with the calculated result.

3. Show, by drawing, how it is possible to draw a line $1\frac{2}{7}$ ins. long, using a ruler graduated in inches and tenths of an inch only.

4. Draw a line 4·8 cm. in length and divide it in the ratio 2 : 3 : 1. Measure the actual lengths obtained and check the accuracy of the construction.

5. Show how to determine the position of points C and D in a straight line AB, 2·7 ins. in length if AC : CD : DB$=$2 : 3 : 4.

6. Construct any triangle XYZ such that $\angle$ZXY$=80^\circ$ and $\angle$XYZ$=30^\circ$. Without measuring determine which is the longest and which is the shortest side.

7. Given that the exterior base angle of an isosceles triangle is 135°, show how to construct the triangle on a base of 2 ins.

8. Show why the sum of the exterior angles of a triangle must equal 4 right angles.

9. A field is in the shape of a rhomboid. One of its sides is 66 yds. long and its area is 1650 sq. yds. A rectangular field

of equal area adjoins it at its 66 yds. side. How wide is the second field ?

10. Show by diagram why a square and a rhombus on the same base cannot be equal in area.

11. A piece of linoleum is in the form of an equilateral triangle of side 10 yds. It is cut up and rearranged to cover a rectangular room 10 yds. in length. Find, by drawing and measurement, the width of the room. (Scale 1 in.=5 yds.)

12. Construct a triangle ABC such that AB=2·3 ins., ∠BAC=80°, ∠ABC=60°. On AB construct a rectangle equal in area to the triangle.

13. Show, by diagram, that the area of a triangle having a base=4 ins. and an altitude =8 ins. is equal to the area of a square on the same base. (Scale $\frac{1}{2}$.)

14. Construct any rectangle ABCD having X the mid-point of DA and Y the mid-point of BC. Join XY and AC. What may be said of the area of XYCD and the area of ABC ? Why ?

15. Show how to draw a rectangle and a triangle on the same base so that the area of the rectangle is $\frac{1}{3}$ that of the triangle.

Quadrilaterals

Any plane figure bounded by four straight lines is a **Quadrilateral.**

Special Quadrilaterals

(i) Any quadrilateral having its opposite sides equal and parallel is a **parallelogram** (see Nos. I, II, III, IV, fig. 29).

(ii) A right-angled and equal-sided parallelogram is a **square** (see No. I, fig. 29).

I II III IV V VI

Fig. 29.

(iii) A right-angled parallelogram not having equal sides is a **rectangle** or **oblong** (see No. II, fig. 29).

(iv) An equal-sided parallelogram, not right-angled, is a **rhombus** (see No. III, fig. 29).

(v) A parallelogram not equal-sided and not right-angled is a **rhomboid** (see No. IV, fig. 29).

(vi) A quadrilateral having one pair only of opposite sides parallel is a **trapezoid** (see No. V, fig. 29).

(vii) A quadrilateral having two pairs only of its adjacent sides equal and also one pair only of its opposite angles equal is a **kite** (see No. VI, fig. 29).

EXERCISE 62

1. Draw any quadrilateral ABCD and copy it. Label the second quadrilateral A′B′C′D′. Using tracing paper, show that ABCD is equal in all respects to A′B′C′D′.

2. Draw a quadrilateral ABCD having AB=2·7 ins., AD=3·2 ins., BC=3·2 ins., and $\angle$BAD=30° and $\angle$ABC=150°. Name the quadrilateral and test its properties.

3. Construct an equilateral triangle OMN on a base OM, 3·9 cm. On OM construct an isosceles triangle OML with base angles=50° and having the apex on the opposite side of OM from N. Name the figure MNO and test its properties.

4. Draw any triangle ABC, and through a point D in AC draw a line parallel to AB cutting BC in E. Name the figure ABED.

5. Draw any quadrilateral ABCD and make a copy of it by the following method. Copy triangle ABD; complete the quadrilateral by copying the triangle BDC.

6. Draw a circle 1·8 ins. radius; take any points A, B, C, D in its circumference; join them to form a quadrilateral. Such a quadrilateral formed in a circle is called a *cyclic quadrilateral. What is the sum of each pair of opposite angles?*

7. Construct a cyclic quadrilateral ABCD and produce AB to E. Measure $\angle$CBE and $\angle$ADC. How may the result be explained?

8. Draw a circle with a radius 1·6 ins., having its centre at O. Take points A, B, C, D in the circumference. Join the points to form a quadrilateral ABCD, and join OA and OC. Show by measurement that $\angle$ADC+$\angle$ABC=half the sum of the angles formed by AO and CO, *i.e.*=2 right angles.

The Area of a Quadrilateral (fig. 30).

The diagonal AC divides quadrilateral ABCD into triangles ABC and ACD, and BP and DO are perpendiculars on AC.

Area of Triangle	$=\frac{1}{2}$ Base $\times$ Altitude.
$\therefore$ Area of ΔABC	$=\frac{1}{2}$ AC $\times$ BP.
and Area of ΔACD	$=\frac{1}{2}$ AC $\times$ DO.
$\therefore$ Area of ABCD	$=\frac{1}{2}$ AC $\times$ BP $+\frac{1}{2}$ AC $\times$ DO
	$=\frac{1}{2}$ AC(BP+DO).

Write the rule in your own words.

The Area of a Trapezoid (fig. 31).

The diagonal AC divides the trapezoid ABCD into triangles ABC and ACD. Each triangle has an altitude $=h$.

$\therefore$ Area of ABCD $=\frac{1}{2}(AB+DC)\times h$.

Write the rule in your own words.

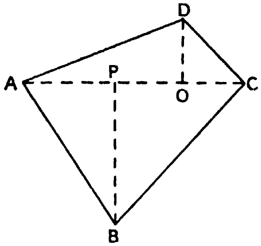


Fig. 30.

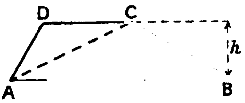


Fig. 31.

The Area of a Kite (fig. 32).

The diagonal AC divides the kite ABCD into triangles ABC and ACD. The perpendiculars from B and D upon AC meet at O.

$$\therefore \text{ Area of ABCD} = \tfrac{1}{2}\, AC(BO+DO)$$
$$= \tfrac{1}{2}\, AC \times BD.$$

Write the rule in your own words.

Reduction of a Quadrilateral to an equivalent Triangle.

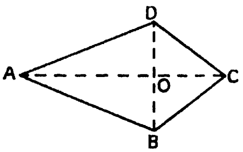


Fig. 32.

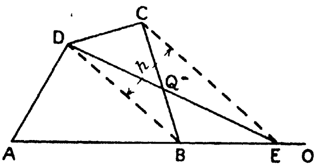


Fig. 33.

ABCD (fig. 33) is the given quadrilateral.

Construction.—Draw diagonal DB. Produce AB to O. From C draw CE parallel to DB, cutting AO in E. Join DE, cutting BC in Q. Then ΔAED is equal in area to quadrilateral ABCD.

Proof: Quadrilateral ABCD $=\Delta$ABD $+\Delta$BCD.
Triangle AED $=\Delta$ABD $+\Delta$BED.

And $\Delta BCD = \Delta BED$, because both have the same base BD and the same altitude (h).

$\therefore$ Triangle BED = Quadrilateral ABCD.

Exercise 63

1. Draw a quadrilateral ABCD having

AB = 8·7 cm., $\angle BAD = 72°$, AD = 5·8 cm., $\angle ABC = 53°$, and BC = 5·2 cm.

Find the area by (i) reduction to an equivalent triangle, and (ii) measurement and calculation.

2. Prove that the area of a trapezoid is equal to the product of the average length of the parallel sides and the altitude of the trapezoid.

3. A piece of zinc is in the form of a trapezoid. The two parallel sides measure 18·6 cm. and 8·4 cm. respectively, and the perpendicular distance between them is 4 cm. Find the value of the zinc at $1\frac{1}{2}$d. per sq. cm.

4. Draw a trapezoid having its parallel sides 4 ins. and 2 ins. respectively and area 9 sq. ins. If a third side of the trapezoid is $3\frac{1}{4}$ ins. long, find by construction and measurement the length of the fourth side.

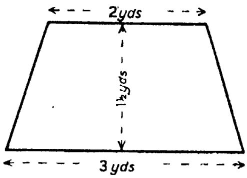


Fig. 34.

5. Fig. 34 represents the cross-section of a tunnel. Find the volume of air enclosed in the tunnel if it is $12\frac{1}{2}$ yds. long.

6. Find the area of the piece of cardboard represented by the trapezoid ABCD (fig. 35).

7. Draw an equilateral triangle on a base of 3·4 ins. Draw its altitude and through its mid-point draw a line parallel to the base. What fraction of the area of the original triangle is the area of the trapezoid formed ?

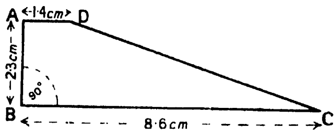


Fig. 35.

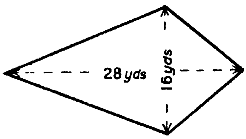


Fig. 36.

8. A kite has diagonals 5 ft. and 12 ft. long. Make one scale drawing of the kite (1 in. = 2 ft.) and calculate its area. How many such drawings can be made? Can the area vary?

9. Find the area of a quadrangle of the dimensions shown in fig. 36.

10. A boy makes the wooden skeleton of his kite by means of two sticks 18 ins. and 9 ins. long at right angles to each other. What will be the surface area of the kite when completed?

Circle, Chord, and Tangent

Fig. 37 shows lines drawn within or touching a circle.

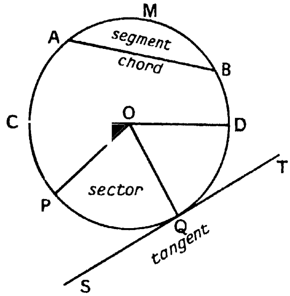


FIG. 37.

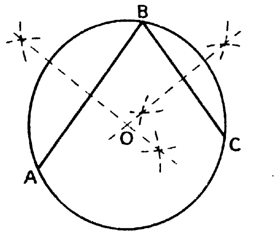


FIG. 38.

A chord is a straight line joining any two points on the circumference, *e.g.* AB.

A diameter is a chord which passes through the centre of the circle, *e.g.* CD. It is the longest possible chord.

A segment is any part of a circle bounded by a chord and an arc: AMB is a minor segment; AQB is a major segment; CMD and CQD are semicircles.

A sector is any part of a circle bounded by two radii and an arc, *e.g.* POQ.

A tangent is a straight line which touches a circle in one point only and does not cut the circle when produced, *e.g.* SQT.

Useful Constructions

1. To find the centre of a given circle.

Take any three points, A, B, C, in the given circle (fig. 38) and draw chords AB and BC. Draw the perpendicular bisectors of

AB and BC and produce them to intersect at O. O is the centre of the given circle.

2. To draw a tangent to a given circle at a given point in the circumference.

Let P be the given point in the circumference of the circle, having its centre O (fig. 39). Join OP. Draw the line RPQ

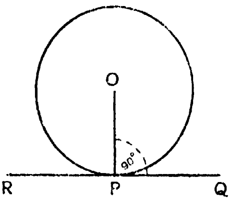


Fig. 39.

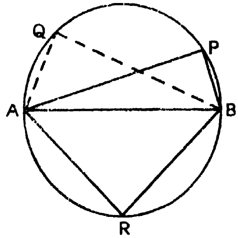


Fig. 40.

perpendicular to OP. RPQ is the required tangent at P, the point of contact.

The angle in a semicircle is a right angle.

Draw a diameter AB (fig. 40), and from points P, Q, and R in the circumference draw lines to A and B. Measure the ∠'s APB, AQB, and ARB. Try as many more as you wish.

The angle in a semicircle is always a right angle.

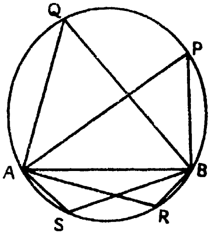


Fig. 41.

The angle in a major segment is acute and the angle in a minor segment is obtuse.

Draw a chord AB (fig. 41) less than a diameter.

In the major segment take points P and Q. In the minor segment take points R and S. Join all these points to A and B. Measure ∠'s APB, AQB, ARB, and ASB.

∠'s APB and AQB are acute and ∠'s ARB and ASB are obtuse. Try as many more as you wish.

Note that angles in the same segment are equal.

3. To draw a tangent to a given circle from a given point outside the circle.

Let P be the given point and O the centre of the given circle (fig. 42). Join PO. Describe the circle of which PO is a diameter and let it cut the given circle in A and B. Join PA and PB, producing both lines. These lines satisfy the required conditions of tangency.

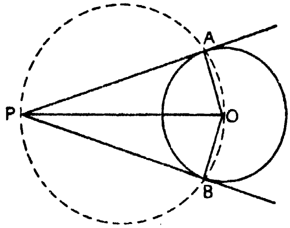


FIG. 42.

EXERCISE 64

1. Take any two points and describe three circles which pass through both points. Where do the centres of the circles lie ?

2. Draw any triangle ABC and construct its circumscribing circle.

3. Draw triangle ABC having AB=5 cm., BC=4 cm., and AC=3 cm. Draw its circumscribing circle. Where is the centre of the circle ? Why ?

4. Draw a line PQ $2\frac{3}{4}$ ins. long and construct $\angle PQR=65°$. Find a point S in QR so that $\angle PSQ$ is a right angle.

5. Construct a rectangle 8 cm. by 3·5 cm. Find a point in one of the longer sides such that when it is joined to the extremities of the opposite side the angle so formed is a right angle. Is there another such point ?

6. Draw a line AB 2·7 ins. long, and on it construct an isosceles right-angled triangle AOB, having the right angle at O.

7. Draw $\angle ABC=90°$, and, using the least possible construction, describe a circle to pass through A, B, and C.

8. In a given circle draw a chord PQ less than the diameter. Take a point A in the major segment and a point B in the minor segment and join to P and Q. Name the figure formed. Measure all its angles and find the sum of the two pairs of opposite angles.

9. Draw a circle of radius 1·25 ins., and at any point T construct a tangent to the circle.

10. Draw a circle of diameter 2 ins. and from a point $2\frac{1}{4}$ ins. away from its centre draw two tangents to the circle.

11. Draw two circles with diameters 2 ins. and $1\frac{1}{2}$ ins. respectively, and having their centres $2\frac{3}{4}$ ins. apart. Construct (*a*) two exterior common tangents, (*b*) two interior common tangents.

12. Draw a circle such that it will pass through two given points A and B and touch a given line PQ.

13. Construct an angle of 60° and draw a circle such that both arms of the angle form tangents to the circle.

14. Construct a rhombus having two angles each $=60°$, and inscribe a circle within it.

15. Construct an isosceles triangle on a base 6·8 cm. long and having an altitude 4·7 cm., and within it inscribe a circle.

16. Inscribe a circle in a square, a rhombus, and a triangle.

17. Construct a triangle having sides 2·8 cm., 3·6 cm., 4·2 cm. long. Produce the sides and construct the three escribed circles.

18. Draw a circle of ·8 in. radius, and fix any point 2·4 ins. from its centre. Now draw a circle to pass through the given point and to touch the given circle. Draw the common tangent to the two circles.

19. Draw a circle with radius 2 ins., and in it draw two chords AB and CD intersecting at Q. Prove by calculation and by diagram that the area of the rectangle $AQ \times QB =$ the area of the rectangle $CQ \times QD$.

20. Draw a circle with radius 2 ins., and draw a diameter AB cutting a chord CD at right angles. Prove, as in question 19, that $AQ \times QB = CQ \times QD = QC^2 = QD^2$. Hence, find, by drawing, the square root of 25; 36; 18; 28.

Polygons

Figures bounded by straight lines are named thus:

(i) **Triangles** when bounded by three straight lines; (ii) **Quadrilaterals** when bounded by four straight lines; (iii) **Polygons** when bounded by more than four straight lines. The various polygons are: Pentagon, having five sides; Hexagon, having six sides; Heptagon, having seven sides; Octagon, having eight sides; and so on.

A polygon with all its sides equal is an **equilateral** polygon (fig. 43).

A polygon with all its angles equal is an **equiangular** polygon (fig. 44).

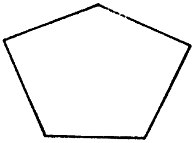


FIG. 43.

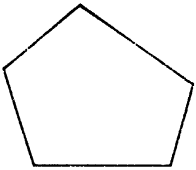


FIG. 44.

A polygon which is both **equilateral and equiangular** is a **regular** polygon (fig. 45).

A polygon having each of its angles less than 180° is a convex polygon, *e.g.* figs. 43, 44, and 45.

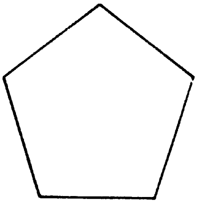


FIG. 45.

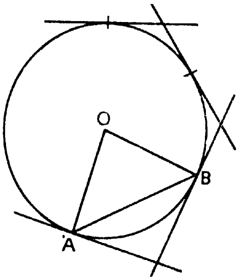


FIG. 46.

The line joining two non-consecutive vertices of a polygon is called a diagonal.

To construct a regular polygon (i) **within,** (ii) **about a given circle.** Draw a circle having its centre O (fig. 46).

(i) At O construct an angle $=\dfrac{360^\circ}{n}$. ($n=$ the number of the sides of the polygon.) Produce the arms of the angle to cut the circum-

ference in A and B. Join AB. n chords, each equal in length to AB, may now be stepped off round the circumference. The resulting figure is the required polygon *within* the given circle.

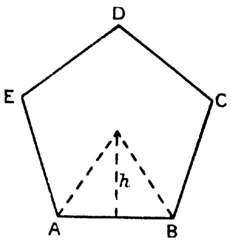


FIG. 47.

(ii) Proceed as in (i) above, and at each vertex of the resulting polygon construct tangents to the given circle. The tangents form the polygon *about* the given circle.

To find the area of a polygon.

Divide it into triangles and find the sum of their areas.

Note that **to find the area of a regular polygon** of n sides (fig. 47), Area $=\frac{1}{2}n \times h \times AB$.

EXERCISE 65

1. Draw a pentagon, a hexagon, and a heptagon. How many sides has each? How many vertices? Show that in a polygon of n sides it is possible to draw $n-3$ diagonals from one vertex of the polygon.

2. In a polygon of n sides the diagonals from one vertex divide the polygon into $n-2$ triangles. Test this statement in the case of a pentagon and hexagon.

3. In a circle having a radius 2 ins. inscribe a regular octagon. Inside the octagon inscribe a circle and measure its diameter.

4. Inscribe a regular hexagon within a circle of $2\frac{1}{2}$ ins. radius. If A, B, C, D, E are five consecutive vertices of the hexagon, join AC, CE, EA, and find without using a protractor the magnitude of $\angle$CAB and $\angle$ACB.

5. Circumscribe a regular hexagon about a circle having a radius $1\frac{1}{2}$ ins. Measure the length of one side and thence calculate the area of the hexagon.

6. In a circle of $1\frac{1}{2}$ ins. diameter inscribe a regular hexagon and also an equilateral triangle. Find, by measurement, the perimeters of the hexagon and triangle.

7. Draw geometrically a regular pentagon, such that its circumscribing circle has a circumference$=3{\cdot}14$ ft. (Scale 2 ins.$=1$ ft.)

8. Construct a regular hexagon of side $1\frac{1}{2}$ ins. Measure the shortest distance from the centre to one of the sides. Find the area of the hexagon.

9. Within a circle having a diameter of 8 cm. inscribe a regular hexagon. Measure the perpendicular distance from the centre of the circle to one of the sides. Find in sq. cm., to two places of decimals, the difference between the areas of the circle and the hexagon.

10. The end of a concertina is hexagonal. The diameter of the end is 9 ins. Draw a dimensioned figure of one end, to a scale 3 ins. = 1 ft.

Surveying

The following tables are used in surveying:

Length.

100 links = 1 chain.
22 yds. = 1 chain.
80 chains = 1 mile.

Area.

$(100 \text{ links})^2 = 10{,}000$ sq. links = 1 sq. ch.
$(22 \text{ yds.})^2 = 484$ sq. yds. = 1 sq. ch.
10 sq. chs. = 100,000 links = 1 acre.

Note.—1 link = 7·92 ins.

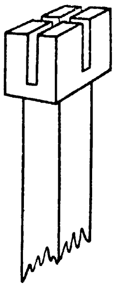


FIG. 48.

When surveying an irregular plot of land a convenient diagonal is drawn and measured. This is known as a **base line** (NS, fig. 49). The perpendiculars to this base line are found by means of a **cross head** (fig. 48), and measured in links and chains. These perpendiculars are called **offsets.** Particulars of the base line and off-sets are entered in a **field note-book,** thus:

Off-sets to Left.	Base Line. North to South.	Off-sets to Right.
	To N.	
	40 chains	
	35 ,,	10 to C.
10 to D	30 ,,	
10 to E	25 ,,	
	20 ,,	5 to B.
	5 ,,	10 to A.
	To S.	

From this data a scale drawing of the irregular polygon repre-

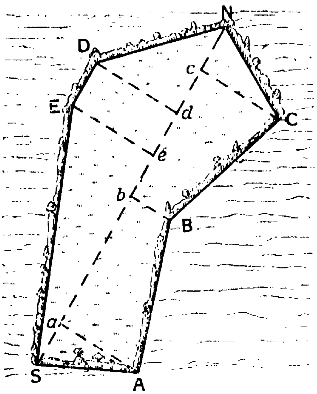


FIG. 49.

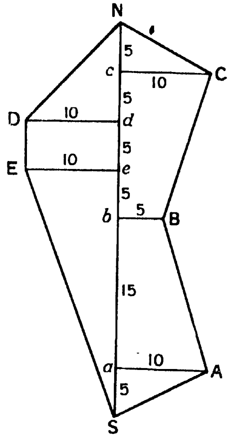


FIG. 50.

senting the area surveyed (fig. 49) is constructed (fig. 50), and from it the required area is calculated.

Total area of polygon SABCNDE is equal to the total area of

triangle SAa	$=\dfrac{5\times 10}{2}$	$= 25$	sq. chains
trapezoid $aABb$	$=\dfrac{(10+5)\times 15}{2}$	$=112\frac{1}{2}$	,, ,,
trapezoid $bBCc$	$=\dfrac{(5+10)\times 15}{2}$	$=112\frac{1}{2}$	,, ,,
triangle cCN	$=\dfrac{5\times 10}{2}$	$= 25$	,, ,,
triangle NDd	$=\dfrac{10\times 10}{2}$	$= 50$	,, ,,
rectangle $DEed$	$=10\times 5$	$= 50$	,, ,,
triangle EeS	$=\dfrac{25\times 10}{2}$	$=125$	,, ,,

$\therefore$ Area of polygon SABCNDE $=500$,, ,,

Exercise 66

From the following extracts from a surveyor's field note-book draw plans of the areas surveyed and calculate their areas.

(1) Scale 1 in. = 100 chains.

Left.	Base Line.	Right.
	To N.	
	350 chains	100 to A
	280 ,,	160 to B
200 to D	150 ,,	
	80 ,,	80 to C
	From S.	

(2) Scale 1 in. = 100 links.

Left.	Base Line.	Right.
	To E.	
	400 links	280 to P
340 to S	340 ,,	
340 to R	200 ,,	
	180 ,,	100 to Q
	From W.	

(3) Scale 1 in. = 100 links.

Left.	Base Line.	Right.
	To W.	
360 to P	380 links	
	320 ,,	140 to M
200 to N	120 ,,	
	From E.	

(4) Scale ½ in. = 10 chains.

Left.	Base Line.	Right.
	To S.	
40 to A	100 chains	
55 to B	78 ,,	
	48 ,,	25 to C
	From E.	

(5) Scale $\frac{1}{10}$ in. = 1 chain.

Left.	Base Line.	Right.
	To N.	
	76 chains	12 to F
41 to A	64 ,,	
	35 ,,	12 to E
	32 ,,	8 to D
20 to B	22 ,,	
	8 ,,	5 to C
	From S.	

(6) Scale $\frac{1}{10}$ in. = 1 yd.

Left.	Base Line.	Right.
	To W.	
18 to A	39 yds.	
	32 ,,	5 to F
10 to B	27 ,,	
	16 ,,	11 to E
8 to C	10 ,,	
	4 ,,	11 to D
	From N.	

Square Root

Example (i). *Find the square root of* 103041.

```
        10'30'41(321
        9
       ----
 62   | 130
      | 124
      |  ----
 641  | ..641
      |   641
      |   ---
      |   ...
```

Steps

1. Mark off the number into periods of two figures each, beginning with the units figure. The last period may contain only one figure. The number of periods is equal to the number of figures in the root.
2. Find the highest number whose square is contained in the last period, and place this number in the root as shown.
3. Subtract the square of 3 from 10.
4. Bring down the next period to the remainder, making 130.
5. Double the root already obtained and place it in the divisor.
6. By trial, determine the rest of the divisor. Place 2 in the root and in the divisor.
7. Subtract 2×62 from 130.
8. Bring down the next period to the remainder, making 641.
9. Continue the above process.

Example (ii). *Find the square root of* 51·9841.

$$\begin{array}{r|l} & 51'{\cdot}98'41(7{\cdot}21 \\ & 49 \\ \hline 142 & \;\;298 \\ & \;\;284 \\ \hline 1441 & \;\;\;1441 \\ & \;\;\;1441 \\ \hline & \;\;\;\;\ldots \end{array}$$

Steps

1. Beginning at the decimal point, mark off the number into periods of two figures, first to the left and then to the right.
2. Continue the processes as in Example (i). Remember to place a decimal point in the root before bringing down the first decimal period.

Compare the above method of finding the square root of a number with the algebraic method (p. 69).

Exercise 67

Find the square roots of :

1. (*a*) 60516. (*b*) 174724. (*c*) 7209225.
2. (*a*) 6·0516. (*b*) 1747·24. (*c*) 7·209225.
3. (*a*) 605·16. (*b*) 17·4724. (*c*) 720·9225.

Evaluate :

4. (*a*) $\sqrt{221841}$ (*b*) $\sqrt{14884}$ (*c*) $\sqrt{17689}$
5. (*a*) $\sqrt{170{\cdot}3025}$ (*b*) $\sqrt{148{\cdot}84}$ (*c*) $\sqrt{176{\cdot}89}$
6. (*a*) $\sqrt{37326{\cdot}24}$ (*b*) $\sqrt{373{\cdot}2624}$ (*c*) $\sqrt{3{\cdot}732624}$

7. If the area of the development of a cube is 144·06 sq. ins., find the length of its edge.

8. The area of a square picture is 49 sq. ins. When framed in a square frame the total area is 179·56 sq. ins. Find (i) the area of the frame alone, and (ii) the length of the edge of the frame.

9. A rectangular garden is 32 ft. $\times$ 512 ft. Find the length of the side of a square garden having its area four times that of the former.

10. What is the perimeter of a square of area 795·24 sq. yds. ?

11. A square floor has an area of 334·89 sq. ft. Find the length of its side.

12. One face of a cube has an area of 39·69 sq. ins. Find its volume.

13. A circle has an area of 300 sq. cm. Find the radius to the nearest $\frac{1}{10}$ millimetre. ($\pi = 3{\cdot}14$.)

14. Find, to three places of decimals, the value of:

$$\text{(i) } \sqrt{26}-\sqrt{2}. \quad \text{(ii) } \sqrt{5}+\sqrt{3}. \quad \text{(iii) } \sqrt{8}+\sqrt{7}.$$

15. A cubical cupboard has a square door having a square window. If the window has an edge 5 ins. and the rest of the door an area of 12·21 sq. ins., find the length of the edge of the door.

16. Find the length of wire netting required to extend four times round a square paddock of area 519,841 sq. ft.

17. Calculate the error, to three places of decimals, when $\sqrt{7}$ is taken as 2·64.

18. What is the length of tape required to bind the edges of a cube of which the outer surface area is 302·46 sq. ins. ?

19. If $a^4 = 118{\cdot}5921$, find the value of a.

The Right-Angled Triangle

Examine the triangle ABC (fig. 51), which is right-angled at B. AB is the base, BC is the altitude, and the side opposite the right angle is the HYPOTENUSE.

Experiment 1.—Draw on squared paper a right-angled triangle, as ABC (fig. 51), having AB 3 divisions long and BC 4 divisions long. Measure the hypotenuse. It is 5 divisions long.

Then *the square on the hypotenuse* (25 small squares) = *the sum of the squares on the other two sides* (9 small squares + 16 small squares).

Experiment 2.—Draw on squared paper a right-angled triangle as before, having AB 5 divisions long and BC 12 divisions long.

Measure the hypotenuse. It is 13 divisions long. Again, the square on the hypotenuse = the sum of the squares on the other two sides.

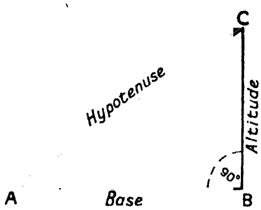


Fig. 51.

Experiment 3.—Draw on squared paper a right-angled triangle as before, choosing your own lengths for the sides. Find again the above relationship of the squares on the sides.

Experiment 4.—Draw any right-angled triangle on cardboard, and then construct squares on the sides. Find the above relationship by weighing.

Experiment 5.—Draw on squared paper a right-angled triangle

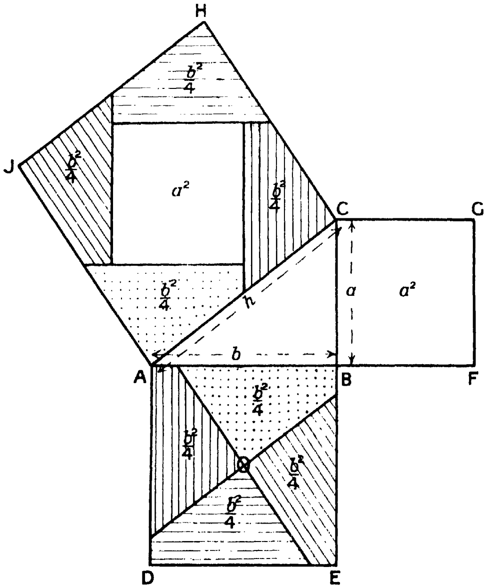


Fig. 52.

ABC, and the squares on each side, as in fig. 52.

O is the mid-point of ADEB.

Through O draw a line parallel to AC, and also through O a line at right angles to AC, thus dividing ADEB into four congruent quadrilaterals.

Cut out these four quadrilaterals and the square CBFG, and arrange them in the positions indicated in fig. 52.

$$\begin{aligned} ACHJ &= ADEB + CBFG. \\ i.e.\quad AC^2 &= AB^2 + BC^2 \\ \therefore\quad h^2 &= b^2 + a^2 \\ b^2 &= h^2 - a^2 \\ a^2 &= h^2 - b^2 \\ \text{And } h &= \sqrt{b^2 + a^2} \\ b &= \sqrt{h^2 - a^2} \\ a &= \sqrt{h^2 - b^2}. \end{aligned}$$

This property of right-angled triangles was first discovered by the Greek, Pythagoras, and is known as the Theorem of Pythagoras. It may be fully stated thus: Whatever figure be constructed on the hypotenuse, the area of it is equal in area to both the similar figures constructed on the other two sides of the right-angled triangle.

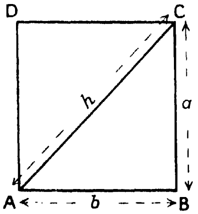


FIG. 53.

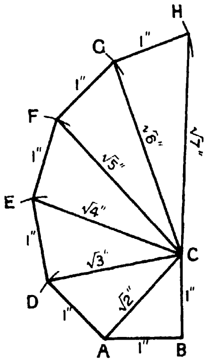


FIG. 54.

Now consider the right-angled triangle ABC, when AB=BC (fig. 53).

The triangle ABC is an isosceles right-angled triangle.

Then $h^2 = a^2 + b^2 = 2a^2$ or $2b^2$.

$\therefore\ h = \sqrt{2a^2} = a\sqrt{2}$.

Complete the square ABCD.

The square on the diagonal = twice the square on one side.

$$\therefore \text{The diagonal of a square} = \sqrt{2} \times \text{side}$$
$$= 1{\cdot}414 \times \text{side}.$$

The properties of right angles suggest a method of finding the square roots of the numbers 1 to 9 by drawing, thus:

Construct an isosceles right-angled triangle ABC having equal sides 1 in. in length (fig. 54). Then AC $= \sqrt{2}''$. Construct right-angled triangle CAD, making AD = 1 in. Then DC $= \sqrt{2+1}$ ins. $= \sqrt{3}''$; similarly EC $= \sqrt{4}''$; FC $= \sqrt{5}''$; and so on. Compare the results obtained by drawing and by calculation.

Exercise 68

1. Two roads meet at right angles, and from the junction two cyclists travel, one on each road. One travels at 16 miles per hour, and the other at 12 miles per hour. What distance is the first cyclist from the other after 15 mins.? Illustrate your answer by a scale drawing of 1 in. = 1 mile.

2. Draw on gummed paper a right-angled triangle, base $= 1\frac{1}{2}$ ins., and altitude = 2 ins. Construct squares on its three sides, and by cutting and superposing show that hypotenuse^2 = base^2 + altitude^2.

3. Construct a square of 7 cm. side, and draw its diagonals and its circumscribing circle. Measure the radius of the circle and check your result arithmetically.

4. The diagonal of a rectangle is 16 cm., and its shorter side is 9·6 cm. Find the length of its longer side. Check your result by constructing the rectangle.

5. The hour hand of a clock is 3·9 cm. long, and the minute hand 5·2 cm. long. Find the distance between the tips of the hands at 9 o'clock.

6. A wireless aerial is stretched from the top of a pole 36 ft. high to the top of a pole 48 ft. high and 38 ft. from the first pole. Find the length of the aerial correct to the nearest $\frac{1}{10}$ ft.

7. The diagonal of a postcard is $6\frac{1}{2}$ ins. One side of the postcard is 3·4 ins. Find (*a*) the length of the adjacent side, and (*b*) the perimeter of the postcard.

8. A ladder 38 ft. 6 ins. long, resting on the ground at a point 5 ft. 10 ins. from a house, just reaches the roof. Find the height of the roof from the ground. If the foot of the ladder is placed 1 yard further away from the house, how much

lower would its other end rest? (Answer to the nearest inch.)

9. Calculate the altitude of a right-angled triangle which has its hypotenuse, $(x+y)$ ins. and its base, $(x-y)$ ins.

10. An isosceles triangle has a base, $2(a+b)$ ins., and its altitude is $\sqrt{3a^2+2ab}$ ins. Find its perimeter.

Volume

We have seen that the volume of any right prism may be calculated from the formula: Volume = Area of Base × Altitude.

Construct of strong paper a cylinder and a cone of equal altitude and equal diameter. Fill the cylinder with sawdust from the cone. The cylinder holds exactly three times as much as the cone.

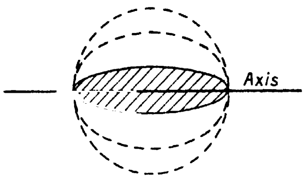


Fig. 55.

Hence the formula for the volume of a cone is

$$\text{Volume} = \frac{\text{Area of Base} \times \text{Altitude}}{3} = \frac{\pi r^2 h}{3}.$$

The volume of a sphere $= \frac{4}{3}\pi r^3$.

When a plane figure revolves about a stationary axis it is said

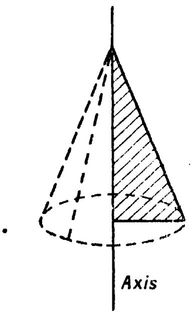


Fig. 56.

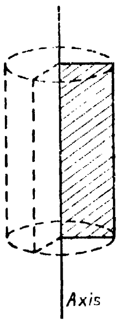


Fig. 57.

to **generate** a solid figure. Thus:

A circle revolving about its diameter generates a sphere (fig 55).

A right-angled triangle revolving about its altitude generates a cone (fig. 56).

A rectangle revolving about one side generates a cylinder (fig. 57).

EXERCISE 69

(In each case, let $\pi = 3 \cdot 14$.)

1. Calculate the cubical content of the solid shown in fig. 58.

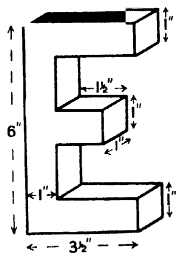


FIG. 58.

2. Find the volume of a cylinder which will just fit inside a cube of side=3 ins. Find also the volume of a cone having the same base and altitude, and of a sphere having the same diameter.

3. A piece of glass tubing 12 cm. long has an external diameter of 7 mm., and an internal diameter of 5 mm. Find the volume of glass it contains.

4. A section of a coping stone is shown in fig. 59. If the stone is $2\frac{3}{4}$ ft. in length, find its total volume.

5. The diagonal of the base of a column of five cubes is 1 ft. 6 ins. Find the volume of the column.

6. A cylindrical pedestal has a volume of 28·5 cu. ft. and is 4·5 ft. in height. Find the area of a horizontal cross-section.

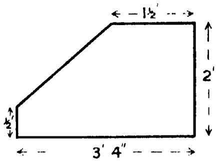


FIG. 59.

7. A cubic foot of copper is drawn into a wire of $\frac{1}{60}$ in. diameter. How long is the wire? (Answer in miles and yards.)

8. How often does the volume of a sphere of 4 ins. radius contain that of a sphere of $1\frac{1}{3}$ ins. radius?

9. A regulation football, when inflated, has a diameter of 28 cm. Find its volume.

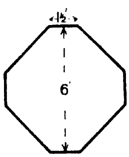


FIG. 60.

10. The base of a supporting stone pillar for a roof is shown in fig. 60. If the pillar is 18 ft. in height, find the total volume of stone it contains.

11. How many cubic inches of lead are required to make 2 yds. of circular lead piping of external diameter $2\frac{1}{2}$ ins. and internal diameter 2 ins.?

12. A boy's hoop has a circumference of 6·5 ft., and it is

made to spin about its diameter. Find the volume of the sphere it generates.

13. An isosceles triangle has a base 2·8 ins. and an altitude 3·7 ins. If it revolves about its perpendicular from apex to base, find the volume of the solid it generates.

14. A conical tent is pitched on a circular patch of ground of radius $1\frac{1}{2}$ yds. If the tent pole is $9\frac{1}{2}$ ft. above the ground, find the air space in the tent.

15. A solid stone monument consists of a cylinder surmounted by a cone of the same diameter. The total height is 28 ft., the height of the cylinder 21 ft., and the diameter 8 ft. Find the total volume of stone in the monument.

16. The mean diameter of a plant pot is 30 cm., and it is 28 cm. deep. How many cubic decimetres of soil will it hold ?

TESTS

Exercise 70a

1. A certain make of motor cycle cost £95 when new, and was later sold second-hand for £70. Calculate the percentage reduction in price correct to the nearest second decimal place.

2. Considering equal volume, copper is 8·65 times as heavy as water. If 1 cu. ft. of water weighs 62·3 lbs., what is the weight of 4·20 cu. ft. of copper ?

3. A roller for a cricket pitch is 3 ft. 3 ins. in diameter. How many turns will it make in travelling a distance of 30 yds. ? Give the answer correct to two places of decimals. ($\pi=3{\cdot}14$.)

4. The minute hand of a clock measures 3·60 ins. from the centre of the clock face to its point. Calculate the area traced out by the minute hand in square inches during one hour. Give the answer correct to the nearest second decimal place. ($\pi=3{\cdot}14$.)

5. (*a*) On a map the distance by rail from Manchester to Bolton is represented by 5·30 ins., and from Bolton to Preston by 10·25 ins. If on the scale of the map $\frac{1}{2}$ in. represents one mile, what is the distance from Manchester to Preston by this railway route ? Give the answer in miles and decimals of a mile.

(*b*) A man earns £3 per week for two weeks, £3, 5s. per week for the next three weeks, and £3, 2s. for the sixth week. Find his average weekly wage for the six weeks.

6. Two flagstaffs are 75 ft. apart, and one is 15 ft. higher than the other. What is the shortest distance from the top of one to

the top of the other? Give the answer in feet and decimals of a foot, correct to two decimal places.

7. A drawing board was made to the following dimensions: length, 30 ins.; width, 24 ins.; and thickness, $\frac{3}{4}$ in. If the wood of which it was made weighs 33 lbs. per cubic foot, calculate the weight of the board in lbs.

8. (*a*) A rectangle has an area of xy sq. ft. and a length of x ins. What is the width of the rectangle in inches?

(*b*) Calculate the value of $\pi(R^2-r^2)$ if $\pi=\frac{22}{7}$, $R=4{\cdot}2$, and $r=2{\cdot}1$.

(*c*) The three sides of a triangle are $a-2b$, $2a+b$, and $3a-4b$ ins. respectively. Give the expression for the perimeter of the triangle in inches.

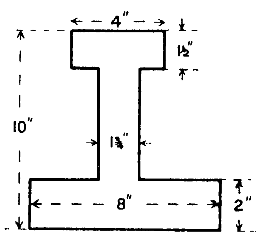


Fig. 61.

9. Fig. 61 represents the cross-section of an iron girder. Find the area of the cross-section, (*a*) in square inches, (*b*) in square feet.

10. (*a*) Solve the equation $\frac{1}{3}(x-7)=31{\cdot}4$.

(*b*) Find the value of V, being given that $S=VT$ and $S=28$ when $T=1{\cdot}25$.

(*c*) What is the value of $\frac{S}{3}(A+4B+2C)$ when $S=1{\cdot}2$, $A=2{\cdot}8$, $B=8{\cdot}4$, and $C=6{\cdot}4$?

Exercise 70b

1. (*a*) Find the value of $(3{\cdot}06+{\cdot}306+306{\cdot}06)\times{\cdot}12$.

(*b*) What is the ratio between $3{\cdot}06$ and ${\cdot}306$?

(*c*) Find the square root of $9{\cdot}3636$.

2. Find the value of x in each of the following equations:

(*a*) $\frac{2}{x}=12$. (*b*) $\frac{a^3x}{b^2}=\frac{a^3}{x}$. (*c*) $3(2x-4)=4-x$.

3. Write down the approximate values of the following lengths, each to the nearest centimetre:

567 mm. 2·63 m. 0·548 m.

4. Simplify:

$$b+\frac{b+c}{b^2-c^2}-\frac{1}{b+c}.$$

In the result you obtain substitute $b=1$, $c=2$, and determine its value.

5. A rectangular plate, breadth 7 ft. 6 in. and thickness $\frac{1}{4}$ in., is made of brass of relative density or specific gravity 8·4. Its weight is $656\frac{1}{4}$ lbs. Find its length. (Assume that 1 cu. ft. of water weighs 62·5 lbs.)

6. (*a*) What is 5 per cent. of 1 ton ? (Answer in lbs.)

(*b*) What weight in grams is equivalent to ·2 per cent. of 1 kilogram ?

(*c*) Find the number of which 20 is 40 per cent.

7. A square has an area of (a^2-6a+9) sq. ft. State the length of its side.

If the length of this side be diminished by 1 ft., find the area of the square formed on it.

8. If two pulleys of diameters D_1 and D_2 are connected by a belt, the respective speeds N_1 and N_2 of the pulleys are given by the expression :

$$D_1 : D_2 = N_2 : N_1.$$

If the pulleys be 18 ins. and 25 ins. diameter, and the smaller pulley makes 125 revolutions per minute, find the revolutions per minute of the larger one.

9. Simplify:

$$\{4x-(x+y)+2(-x+y)\}\{x(x-y)+y^2\}.$$

10. By using a screw jack, a weight R is just raised by a force P.

R lbs. .	20	30	50	75	100	125
P lbs. .	1·5	2·1	3·1	4·3	5·2	6·2

Show the relation between weight raised and force used by means of a graph, plotting weight raised horizontally to the scale 1 in. = 20 lbs., and force used vertically to the scale 1 in. = 1 lb. From the graph find the force required to raise a weight of 60 lbs., and the weight raised by a force of 6 lbs.

Exercise 70c

1. (*a*) Find the value of $75{\cdot}6 \times {\cdot}085 \div ({\cdot}5 \text{ of } 160)$.

(*b*) What will be the cost of painting 100 ft. of piping $3\frac{1}{2}$ ins. outside diameter at 4d. per square foot ? ($\pi = \frac{22}{7}$.)

(*c*) A uniform bar of iron, length 7 ft. 6 ins., has a weight of 60 lbs. What will be the weight of the remaining piece after a length of 2 ft. has been cut off ?

2. A metal sash weight for a window is to be $2\frac{1}{2}$ ins. square in section, and it must weigh 30 lbs. Find its length if 1 cu. in. of the metal weighs ·48 lb.

3. Given that 1 cu. cm. of copper weighs 8·94 grams, find the weight of 12 metres of cylindrical copper wire of 2 mm. diameter. ($\pi=3{\cdot}14$.)

4. AB and BC (fig. 62) represent two roads at right angles to each other. From A to C there is a straight path. What distance is saved by taking the path from A to C, instead of walking along AB and BC ?

5. Find the cost, at $4\frac{1}{2}$d. per lb., of the piece of mild steel plate

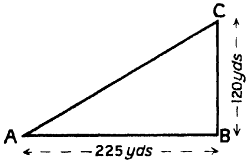


Fig. 62.

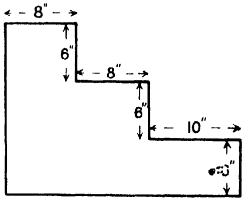


Fig. 63.

shown in fig. 63, if 1 sq. ft. weighs 5·6 lbs.

6. In a friction experiment with a weighted slider dragged along a horizontal surface, the following figures were obtained :

Total weight dragged (lbs.)	2	4	6	8	9
Horizontal pull (lbs.) .	·7	1·4	2·05	2·75	3·1

To the largest scale your paper will allow, plot weight along the longer side of the squared paper and pull at right angles to it. Draw the graph and find from it the pull for a weight of $6\frac{3}{4}$ lbs.

7. (*a*) Find the value of x in the equation :

$$\cdot 4(x+3)=\cdot 5(x-1).$$

(*b*) If the average weight of pulleys is $(3x+2y)$ lbs., find the weight of $x(3x-2y)$ pulleys. Clear your answer of brackets.

8. (*a*) A square tile has an area of $\dfrac{3b+2c}{2c-4b}$ sq. ft.

Given $b=3$, $c=8$, find the length of one edge.

(*b*) The stress in a tie bar is given by $F=\dfrac{W}{A}$ where $A=\dfrac{\pi d^2}{4}$.

Arrange the formula to find the value of W when the values of d and F are given.

9. Fig. 64 represents the plan of a table top with semicircular ends. What is its area in square inches ?

10. Find the value of x in each of the following equations :

$$(x+2)\ (x+3)=x^2+8x-3.$$
$$(x-4)\ (x+3)=x(x-3)-2.$$

Exercise 70d

1. (*a*) Find the value of $(4{\cdot}5-1{\cdot}012)\times4{\cdot}5$.

(*b*) Three pieces of copper wire measure $4\frac{1}{2}$ yds., $4\frac{1}{2}$ ft., and $4\frac{1}{2}$ ins. in length. Give their total length in feet.

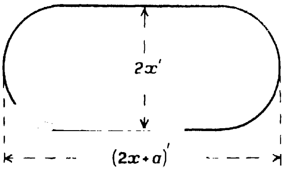


Fig. 64.

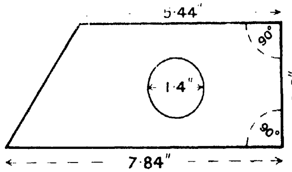


Fig. 65.

2. Thirty-four cu. cm. of a liquid weigh 26·52 grams. What weight of the liquid would just fill a litre flask ?

3. A piece of iron with lengths of sides as shown in fig. 65 has a circular hole 1·4 in. in diameter drilled out of it. Find the area, correct to two decimal places, of the remaining surface. ($\pi=3{\cdot}14$.)

4. Oak is ·765 times as heavy as water, volume for volume. Given that 1 cu. ft. of water weighs 1000 ozs., find the weight of a beam of oak 14 ft. long, 1 ft. 6 ins. wide, and 8 ins. thick. (Answer in lbs.)

5. An open cylindrical cistern, 2 ft. 6 ins. high and 18 ins. in diameter, is supplied with water from a ball tap to within 2 ins. of the top of the cistern. How many gallons of water will there be in it ? (1 gall. = 277 cu. ins.)

6. In an experiment with a spiral spring it is found that each 10 lbs. of weight added extends the spring 1·25 ins. Draw a graph as large as the squared paper will allow, showing extensions of the spring for weights from 0 lbs. to 50 lbs. From your graph find the extension for a weight of 37 lbs.

7. (*a*) The base of a triangle is $(3x-4)$ ins., and the altitude $(2x-2)$ ins. long. What is its area ?

(*b*) The area of a rectangle is $(x^2+3x-28)$ sq. ft., and its breadth is $(x-4)$ ft. Find its length in inches.

8. (*a*) The following are the lengths in inches of four bars of iron :

$$3x,\ 4{\cdot}5x,\ 2x,\ x.$$

Their total length is 5 ft. 3 ins. What is the value of x ?

(b) $3+\frac{x}{\cdot 5}=7-x$. Find what x is equal to.

9. (a) If $H=1\frac{1}{2}$ and $L=2$, find the value of R when $R=\frac{L^2}{6H}+\frac{H}{2}$.

(b) Rearrange the formula $S=Vt-\frac{1}{2}gt^2$ so as to obtain the value of V when the other quantities are known.

10. Find the value of the following to two places of decimals:

(a) $3\cdot1416\times5\cdot61$.

(b) $25\cdot08\div0\cdot7854$.

Exercise 70E

1. (a) Find the value of $\frac{10\cdot8}{59\cdot2}\times\left(\frac{1\cdot35}{2\cdot7}+\frac{3}{32\cdot4}\right)$.

Give your answer to four places of decimals.

(b) During three months 85,772 vessels were convoyed across the ocean, and 433 were lost through enemy action. What was the percentage loss ?

Give your answer to two places of decimals.

2. (a) If mild steel weighs 490 lbs. per cubic foot, what is its weight per cubic inch ?

Give your answer in lbs. to three places of decimals.

(b) Given that 1 metre=39·37 ins., find, to the nearest yard, how many more yards there are in 1 mile than in 1 km.

3. Fig. 66 represents a piece of 7 lb. lead (lead weighing 7 lbs. per square foot). What is the weight of the piece ?

4. The total surface area of a cube is 346·56 sq. ins. Find the length of one edge.

5. A swimming bath, 36 ft. 6 ins. long and 16 ft. wide, is filled with water which is 3 ft. 6 ins. deep at one end and 5 ft. 6 ins. deep at the other. How many cubic feet of water does the bath contain ?

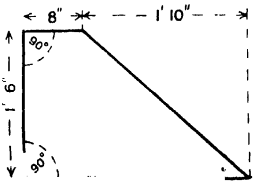


Fig. 66.

6.

Load in lbs.	0	7	14	21	28	42	56
Effort in lbs.	3	4·75	6·5	8·25	10	13·5	17

The above figures were obtained from experiments with wooden blocks and tackle. Draw a graph to show the connection between load and effort. Plot load from left to right along the longer side of the squared paper to a scale of 1 in. to 5 lbs., and effort at right angles to a scale of 1 in. to $2\frac{1}{2}$ lbs. From the graph find the effort when the load is 50·5 lbs.

7. (*a*) The lengths in yards of the three sides of a triangle are $a+b-c$, $a-(b+c)$, and $a-b-c$. Find its perimeter in feet.

(*b*) $T=\dfrac{\pi f d^3}{16}$. Obtain the value of f when $T=33{,}000$, $d=4$, and $\pi=\frac{22}{7}$.

8. A rectangular piece of tinplate and a square piece have both the same area, but the length of the rectangular piece is longer by 16 ins., and its width shorter by 8 ins., than the side of the square piece. What are the dimensions of both pieces ?

9. (*a*) Find the value of x in the equation $\dfrac{3x-2}{4}-\dfrac{2x}{5}+2=0$.

(*b*) A cylindrical column of stone has a volume of (x^2+x-12) cu. ft. and a height of $(x+4)$ ft. What is the area of the base ?

(*c*) An iron girder is $(4x^2-2xy+y^2)$ yds. long ; (x^2-3y^2) yds. are cut off. What is the length in yards of the remaining piece ?

10. Simplify :

(*a*) $\dfrac{a^2-b^2}{a+b}$. (*b*) $\dfrac{a^2+2ab+b^2}{a+b}$. (*c*) $\dfrac{a^2-ac+ab-bc}{a(a+b)}$.

Exercise 70F

1. (*a*) Find the value of $\dfrac{16{\cdot}25-10{\cdot}6}{\sqrt{112{\cdot}36}}$.

(*b*) A triangle has an area of $4\frac{7}{18}$ sq. ft. ; its base is 6 ft. 7 ins. long. Find its altitude.

(*c*) A metal cylinder has a diameter of 3 ins. and a length of 9 ins. What is the ratio between the area of one of its circular ends and the area of its curved surface ?

2. A rough casting supplied from a foundry weighed 2 cwts. 2 qrs. 14 lbs., and after being turned in the lathe it weighed 2 cwts. 21 lbs. What was the percentage of the finished weight removed in the process of turning ?

3. A brass cone, with a perpendicular height of 5 ins. and diameter of circular end $5\frac{1}{4}$ ins., is placed into a vessel having each of its inside surfaces square. The circular end of the cone when

lying flat is found to touch the four upright sides. What volume of water must now be added to fill the vessel ?

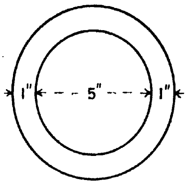


Fig. 67.

4. Fig. 67 shows the cross-section of a cast-iron pipe. If cast iron weighs 450 lbs. per cubic foot, find the weight of a 6 ft. length of the pipe. ($\pi = \frac{22}{7}$).

5. A cable weighs 2·706 kilograms per metre. How many grams per centimetre is this ? Find, to the nearest yard, the length of cable weighing 3600 kilograms. (1 metre = 39·37 ins.)

6. A copper wire was loaded gradually, with the following results :

Load in lbs.	0	4	8	12	16	18	20	23
Increase in length in inches	0	·02	·03	·04	·06	·07	·08	·10

Draw a graph to show the relation between these two sets of quantities, plotting load to the scale of 1 in. to 2 lbs. along the longer side of the squared paper, and increase in length at right angles to a scale of 1 in. to ·02 in.

From the graph find the probable increase in length when the load was increased from 3 lbs. to $17\frac{1}{2}$ lbs.

7. (*a*) What must be added to the sum of $2x^2 - 3xy + 3y^2$, $2xy + y^2$, and $-(2x^2 - xy - y^2)$ to make 4 ?

(*b*) The formula $F = \frac{Mv^2}{gr}$ is arranged for finding the value of F when the other quantities are known. Rearrange it for finding the value of v.

(*c*) A man walks for t hrs. at m miles per hour. If he returns the same way at 3 miles an hour, resting for x mins., how many hours will the journey back take him ?

8. (*a*) $\frac{x-1}{2} - \frac{2x-7}{3} = 2x - 1$. Find the value of x.

(*b*) Given $\frac{F-32}{9} = \frac{C}{5}$; find F if $C = -10$.

(*c*) Simplify $\frac{x^2 - 4y^2}{xy(x+2y)^2} \div \frac{2x-4y}{x^3y^3}$.